Value-Added and Productivity Linkages Across Countries*

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Abstract

Traditional international real business cycle models produce a weak relationship between trade and cross-country real GDP correlations, contradicting empirical findings. This puzzle can be resolved by defining real GDP in the model using double deflation and base period prices, mirroring how the data is constructed. Whenever imported input's base period price does not reflect their marginal revenue product, real GDP movements become mechanically linked to fluctuations in imported inputs. Focusing on the cases of markups and love of variety, we quantitatively show that input trade is associated with the synchronization of real GDPs, measured productivities and profits, consistent with data.

Keywords: International Trade, Comovement Puzzle, Real GDP Measurement, Networks, Input-Output Linkages, Solow Residual.

JEL Classification: F12, F44, F62

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1 Introduction

Since the seminal contribution of Frankel and Rose (1998), the role of trade in propagating shocks across countries has been the subject of substantial empirical attention. Two countries with stronger trade linkages tend to experience more synchronized business cycles.[1] Yet, Kose and Yi (2001, 2006) has found that traditional international real business cycle (IRBC) models are unable to quantitatively account for the empirical relationship by an order of magnitude. This failure of standard models is referred to as the *Trade Comovement Puzzle* (TCP) and remains an important open question in international macroeconomics.[2]

This paper argues that the discrepancy between the empirical association and standard models' predictions can be reconciled by improving the mapping between macroeconomic models and the data. It is well known that the real GDP measured by statistical agencies differs from the "real value added" commonly used in many macroeconomic models. However, we show that this difference becomes particularly significant in the presence of both input-output linkages and price distortions, and that the TCP is a byproduct of this measurement issue.

To articulate our main point, we introduce two real value added concepts: *Physical Value Added* (PVA) and *statistical value added*. The former is often used in IRBC models, while the latter captures what is actually measured by statistical agencies when they construct real GDP. Physical Value Added describes the "net physical output" of an industry and has roots in Fabricant (1940), Sims (1969), and Arrow (1974). Consider a country that produces its gross output by combining domestic input A and imported input B. The Sims (1969)-inspired PVA definition of real value added is based solely on the physical quantity of domestic input A. By construction, changes in imported input B impact PVA only insofar as it changes the quantity of input A. As a result, standard IRBC models that use the PVA definition generate a weak link between foreign input usage and fluctuations in real value added.

In reality, however, statistical agencies do not observe Physical Value Added and construct a measure of *statistical value added* – real GDP in national accounts databases – using double deflation and base period prices. This procedure involves taking the difference between gross output and intermediate inputs, both valued using base period prices. Our core argument is that real GDP, as measured by statistical agencies, is equal to physical value added only when

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[1] For empirical studies supporting the association between international trade and business cycle synchronization, see Clark and van Wincoop (2001), Imbs (2004), Baxter and Kouparitsas (2005), Kose and Yi (2006), Calderon et al. (2007), Inklaar et al. (2008), Di Giovanni and Levchenko (2010), Ng (2010), Liao and Santacreu (2015), Duval et al. (2015), Di Giovanni et al. (2018), Avila-Montealegre and Mix (2020), and Drozd et al. (2021).

[2] For quantitative studies of the TCP, see for instance Kose and Yi (2001, 2006), Burstein et al. (2008), Arkolakis and Ramanarayanan (2009), Johnson (2014), Liao and Santacreu (2015), and Avila-Montealegre and Mix (2020).

the base period prices used in its construction are not distorted. That is, if the base period price used for intermediate inputs reflects both their marginal cost and the marginal revenue that can be derived from their use. If, instead, the base period price used in the valuation of imported inputs is lower than the marginal revenue product of these inputs, then an increase in foreign input usage is associated with higher *statistical value added* (i.e. higher real GDP), even if domestic factors and technology remain constant. In such case, real GDP and physical value added fluctuations diverge.

To be clear from the outset, we do not wish to compare all possible mechanisms through which shocks can propagate across countries. Instead, our goal is to highlight and clarify the role of base period prices, and to show that any distortion between the price used for valuing imported input and their marginal revenue product creates a link between input usage and the growth rate of real GDP, thereby increasing the strength of shock transmission between countries. Specifically, we focus on two common sources of such a distortion in the macroeconomic and trade literature, markups and love of variety. While the inclusion of input-output linkages together with price distortions has been examined in the literature, we show that it is the combination of these elements with a statistically-consistent real GDP measurement (using double deflation and base period prices) that generates a strong Trade Comovement slope (hereafter, TC slope).

Using a simple theoretical framework, we demonstrate that accounting for real GDP fluctuations caused by changes in imported inputs in the presence of markups and love of variety is crucial for establishing the relationship between trade linkages and business cycle synchronization. With markups, which imply non-zero profits in the domestic economy, intermediate inputs generate more revenues than their cost. Therefore, using more imported inputs leads to higher real GDP, even when domestic factors and technology are unchanged. Importantly, our argument does not require variable or heterogeneous markups – the mere existence of constant markups in the base period prices used to calculate real GDP mechanically links domestic real GDP to fluctuations in imported input usage. When there is love of variety, accessing a wider range of foreign inputs leads to efficiency gains that are not reflected in imported input prices. Again, using more imported inputs leads to higher real GDP, over and beyond any change in domestic factors or technology. As a result, measured productivity is also directly affected by foreign shocks.

We then quantify our theory and demonstrate that constructing real GDP following the procedure used by statistical agencies helps reconcile theory and empirical findings. To test this, we build on the standard IRBC model and extend it with monopolistic competition and firms' entry and exit. We calibrate the model with 35 countries that represent 91% of world GDP and 64% of world trade flows. Keeping technology shocks unchanged, we use variations of trade flows between 1970 and 2009 to assess the ability of the model to produce a strong TC slope. We compare our quantitative results to the data using a panel composed of the same countries, which enables the inclusion of dyadic and time-window fixed effects in the estimation. We find that the correlation between trade and GDP comovement is primarily driven by *trade in intermediate inputs*, while the effect of trade in final goods is not significant or even negative. This robust finding is consistent with our theory and further confirmed in our quantitative model. When real GDP is correctly constructed, our model reproduces 75% the observed *trade-comovement (TC) slope*. As a result, the theory delivers an important channel to resolve the Trade-Comovement Puzzle. As expected, the correlation between input trade and the synchronization of physical value added is significantly lower in our simulations, with the slope being less than one-tenth of that obtained with real GDP.

Finally, we provide additional evidence supporting the mechanism puts forward in this paper. As changes in imported inputs impact real GDP beyond changes in domestic factor supply or technology, the Solow Residual is directly affected by foreign shocks. We show that this prediction is supported in our model as well as in the data: higher input trade is associated with an increase in the synchronization of aggregate profits and the Solow Residual. Additionally, we find that higher business cycle synchronization is associated with movements in the number of traded varieties, with both the range and the variance of extensive margin fluctuations being associated with a surge in GDP correlation.

Related Literature. Our work builds on a number of previous papers that helped refine our understanding of the relationship between bilateral trade and GDP comovement. On the empirical side, since the empirical contribution of Frankel and Rose (1998), many studies have confirmed the positive association between trade and real GDP synchronization. On the theoretical side, many papers have refined and highlighted possible ingredients to solve the puzzle initiated by Kose and Yi (2001). We believe a combination of multiple factors can help standard macro models to fully reproduce a high TC slope, and recognize that the mechanism we describe in this paper is not the only explanation at play.

Burstein et al. (2008) show that allowing for international production sharing can deliver tighter business cycle correlation if the elasticity of substitution between home and foreign intermediate inputs is extremely low. Arkolakis and Ramanarayanan (2009) analyze the impact of vertical specialization on the relationship between trade and business cycle synchronization. Their model with perfect competition does not generate significant dependence of business cycle synchronization on trade intensity, but they show that the introduction of time-varying price distortions improves the model's fit with the data. Incorporating trade in inputs in an otherwise standard multi-country IRBC model, Johnson (2014) shows that input-output linkages *alone* are not sufficient to solve the trade-comovement puzzle. Nevertheless, his work points that while such production linkages do not synchronize real GDP, they do generate comovement in gross output as well as input usage. The three papers above feature perfectly competitive models in which real GDP is measured as "physical value added". Compared to them, we add monopolistic competition and firms' entry and exit, and argue that once real GDP is measured using double deflation and base period prices, those ingredients reconcile models' predictions and the data regarding the TC slope.

The role of markups in generating a link between intermediate input and measured productivity has been discussed in several papers such as Hall (1988) and Basu and Fernald (2002), and more recently in Gopinath and Neiman (2014). With markups, intermediate inputs generate more revenues than their cost. Hence, statistical real value added can be created by simply using more inputs, even with fixed domestic factors and technology. The importance of love of variety and fluctuations in the number of imported varieties has been pioneered by Feenstra (1994). Most related to our international comovement focus, Liao and Santacreu (2015) uses a two-country model to show that when measured productivity is scaled by the number of varieties, a country-specific shock generates cross-country TFP comovement from its effects on firms' entry and exit. In comparison to their work, this paper highlights and clarifies the significance of accurate measurement of real GDP. We demonstrate that while incorporating price distortions and/or extensive margin adjustments may significantly impact the fluctuation of real GDP as measured in the data, it does not materially change the model's propagation property if one looks at physical value added.

Additionally, two recent works are worth discussing. In a complementary approach, Drozd et al. (2021) shed lights on a key model ingredients that enable to generate a link between international trade and factor supply synchronization. They show that the introduction of dynamic trade elasticity in the presence of convex capital adjustment costs can account for around 70% of the TC slope. Finally, note that the measurement issue discussed here is at work with homogenous markups, which is in contrast to Baqaee and Farhi (2020) who examine the misallocation of resources across heterogeneous firms.

2 Theory: On the Definition and Measurement of Real GDP

We begin by showing that real GDP, as measured by statistical agencies, differs from the theory-consistent definition of real value added. Specifically, when a country imports intermediate inputs at a price that does not reflect their marginal revenue product, the usual statistical definition of real GDP does not equal the net physical production of the economy. In this case, real GDP can be expressed as the sum of a first term that captures the physical value added and additional terms that capture the difference between the marginal cost of imported inputs and the marginal revenue they generate. In addition, this measurement issue creates a gap between measured productivity and actual technology.

2.1 A Simple Accounting Framework

Consider an economy that produces a gross output (GO) using domestic factors (K, L) and imported inputs (X). According to Sims (1969) and Arrow (1974), if the production function for gross output is separable between primary factors and imported inputs, real value added can be defined implicitly from the production function itself. If $GO = Q(K, L, X)$ can be re-written as $GO = Q(V(K, L), X)$, then we can "*imagine capital and labor cooperating to produce an intermediate good called real value added (V), which in turn cooperates with materials to produce the final product*" (Arrow (1974), pp 4-5). Using this definition, real value added can be thought of as "physical value added" and its fluctuations are only attributed to changes of the value added bundle $V(K, L)$.

In practice, real value added is measured using double-deflation as the difference between gross output and intermediate inputs, both valued using base period prices. As a result, real GDP equals physical value added if and only if the base period price used for intermediate inputs reflects both their cost and the marginal revenue that can be derived from their usage.

We focus on two widely-used ingredients that create a wedge between imported inputs' base period price and their marginal revenue product: markups or/and love of variety. With these features, we show that real GDP fluctuations, as measured in the data, are not limited to changes of the theory-consistent physical value added, but are also the direct result of changes in the quantity and variety of imported input. By creating a mechanical link between real GDP and imports, those features allow for a quantitative resolution of the TCP.

Setup. Consider an economy with N countries. Gross output in country i is produced using domestic factors ($L_{i,t}$ and $K_{i,t}$) and imported inputs ($X_{i,t}$) according to:

$$GO_{i,t} = \Big[\underbrace{Z_{i,t} L_{i,t}^{\alpha} K_{i,t}^{1-\alpha}}_{\text{Physical Value Added}} \Big]^{\gamma} \cdot \Big[\underbrace{X_{i,t}}_{\text{Imported Inputs}} \Big]^{1-\gamma}, \tag{1}$$

where γ is the value added share of gross output and $Z_{i,t}$ is value-added TFP.

Markup. Let $\mu_{i,t}$ be the ratio between sales (i.e. Gross Output valued at current price $P_{i,t}$) and total cost ($TC_{i,t}$) in country i, defined as:

$$\mu_{i,t} = \frac{P_{i,t} GO_{i,t}}{TC_{i,t}}. \tag{2}$$

There are many reasons why $\mu_{i,t}$ could be above one. For example, monopoly power could allow gross output price to be above its marginal cost. Alternatively, any tax collected on value added and passed on prices would also imply $\mu_{i,t} > 1$.

Extensive Margin. We introduce love of variety in gross output production in the form of a Dixit-Stiglitz aggregation of many varieties of imported inputs. Let the imported input bundle $X_{i,t}$ be a CES aggregate of $M_{i,t}$ varieties, such that:

$$X_{i,t} = \left(\int_{0}^{M_{i,t}} x_s^{\frac{\sigma-1}{\sigma}} ds \right)^{\frac{\sigma}{\sigma-1}}. \tag{3}$$

Assuming foreign producers are symmetric and denoting by $x_{i,t}$ their (common) production level, $X_{i,t}$ reduces to $X_{i,t} = M_{i,t}^{\sigma/(\sigma-1)} x_{i,t} = M_{i,t}^{1/(\sigma-1)} \cdot M_{i,t} x_{i,t}$. Moreover, denoting by $p_{i,t}^{x}$ the (common) price of a given variety, the ideal price index dual to the CES aggregation is given by $\mathcal{P}_{i,t} = M_{i,t}^{1/(1-\sigma)} \cdot p_{i,t}^{x}$. Denoting $\widehat{Y}_t = \frac{\Delta Y_t}{Y_{t-1}} \approx d\ln(Y_t)$ the proportional change of any variable Y, changes in the imported input bundle can be expressed as:

$$\widehat{X}_{i,t} = \underbrace{\widehat{M_{i,t} x_{i,t}}}_{\text{Change in total imports}} + \underbrace{\frac{1}{\sigma - 1} \widehat{M}_{i,t}}_{\text{Entry/Exit Effect}}, \tag{4}$$

In equation (4), the first term is the physical change in total imported inputs, while the second term measures the variation in $X_{i,t}$ due to changes in the number of available varieties. As discussed in Feenstra (1994), when the production function exhibits love of variety, any increase in the mass of input suppliers leads to a surge in efficiency. As we will see, this channel amplifies the quantitative impact of imported input movements on (measured) real GDP fluctuations.

Value Added. We define two concepts of real value added in this economy. First, in line with Sims (1969) and others, we define *Physical Value Added* (PVA) implicitly from the production function as $PVA_{i,t} = Z_{i,t} L_{i,t}^{\alpha} K_{i,t}^{1-\alpha}$. Second, we follow the procedure used by statistical agencies and construct real GDP (RGDP) using double deflation and base period prices. More precisely, RGDP growth between $t-1$ and t is constructed by valuing quantity changes with $t-1$ prices. Proportional changes of the two real value added indices can be expressed as:[3],[4]

$$\text{Physical Value Added :} \quad \widehat{PVA}_{i,t} = \widehat{Z}_{i,t} + \alpha \widehat{L}_{i,t} + (1-\alpha)\widehat{K}_{i,t}\,, \tag{5}$$

$$\text{Real GDP :} \quad \widehat{RGDP}_{i,t} = \frac{P_{i,t-1}\Delta GO_{i,t} - p_{i,t-1}^{x}\Delta(M_{i,t}x_{i,t})}{P_{i,t-1}GO_{i,t-1} - p_{i,t-1}^{x}(M_{i,t-1}x_{i,t-1})}\,. \tag{6}$$

Note that, consistent with statistical agencies' procedures, the measurement of imports in equation (6) is based on quantities ($M_{i,t-1}x_{i,t-1}$) and does not incorporate variety effects.

Proposition 1 *Consider a production economy described by* (1) *to* (3) *and two definitions of real value added described by* (5) *and* (6)*. Real GDP and Physical Value Added are related by:*

$$\widehat{RGDP}_{i,t} = \omega_{i,t-1}\left[\gamma \widehat{PVA}_{i,t} + \underbrace{(1-\gamma)}_{\substack{\textit{imported input share}\\ \textit{in domestic production}}} \left(\underbrace{\frac{\mu_{i,t-1}-1}{\mu_{i,t-1}}\widehat{X}_{i,t}}_{\textit{Markup Effect}} + \underbrace{\frac{1}{\mu_{i,t-1}(\sigma-1)}\widehat{M}_{i,t}}_{\textit{Variety Effect}} \right)\right]. \tag{7}$$

Proof: Equation (6) can be written as:

$$\begin{aligned}
\widehat{RGDP}_{i,t} &= \underbrace{\frac{P_{i,t-1}GO_{i,t-1}}{P_{i,t-1}GO_{i,t-1} - p_{i,t-1}^{x}(M_{i,t-1}x_{i,t-1})}}_{=\,\omega_{i,t-1}} \left[\frac{\Delta GO_{i,t}}{GO_{i,t-1}} - \frac{p_{i,t-1}^{x}\Delta(M_{i,t}x_{i,t})}{P_{i,t-1}GO_{i,t-1}}\right] \\
&= \omega_{i,t-1}\left[\gamma\left(\widehat{Z}_{i,t} + \alpha\widehat{L}_{i,t} + (1-\alpha)\widehat{K}_{i,t}\right) + (1-\gamma)\widehat{X}_{i,t} - \frac{p_{i,t-1}^{x}(M_{i,t-1}x_{i,t-1})}{P_{i,t-1}GO_{i,t-1}}\widehat{M_{i,t}x_{i,t}}\right].
\end{aligned}$$

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[3] As discussed in Burstein and Cravino (2015), the BEA does not use $t-1$ prices to construct real GDP, but rather a Fisher chain-weighted price index, according to:

$$\widehat{RGDP}_{i,t} = \left(\frac{P_{i,t-1}GO_{i,t} - p_{i,t-1}^{x}X_{i,t}}{P_{i,t-1}GO_{i,t-1} - p_{i,t-1}^{x}X_{i,t-1}}\right)^{0.5}\left(\frac{P_{i,t}GO_{i,t} - p_{i,t}^{x}X_{i,t}}{P_{i,t}GO_{i,t-1} - p_{i,t}^{x}X_{i,t-1}}\right)^{0.5}.$$

Intuitively, the Fisher index is a geometric average between two base period pricing methods, alternatively using $t-1$ and t prices. We simplify the discussion and use $t-1$ prices, also known as the Laspeyres index.

[4] Equations (5) and (6) are expressed in terms of growth rate (and not in *levels*), which is consistent with all our quantitative results in section 4.2 where real GDP is HP-filtered. In practice, the *level* of RGDP at time t is constructed iteratively using the level at $t-1$ and the the growth rate as defined in (6).

Using the definition of PVA as well as equation (4) then leads to:

$$\begin{aligned} \widehat{RGDP}_{i,t} \quad &= \quad \omega_{i,t-1}\left[\gamma\widehat{PVA}_{i,t} + \left((1-\gamma) - \frac{p^x_{i,t-1}(M_{i,t-1}x_{i,t-1})}{P_{i,t-1}GO_{i,t-1}}\right)\widehat{X}_{i,t}\right. \\ &\qquad \left. + \ \frac{p^x_{i,t-1}(M_{i,t-1}x_{i,t-1})}{P_{i,t-1}GO_{i,t-1}}\frac{1}{\sigma-1}\widehat{M}_{i,t}\right]. \end{aligned} \tag{8}$$

The definition of $\mu_{i,t}$ in equation (2) provides a relationship between the intermediate input share of total cost, $(1-\gamma)$, and the share of total sales, $\frac{p^x_{i,t-1}(M_{i,t-1}x_{i,t-1})}{P_{i,t-1}GO_{i,t-1}}$, such that:

$$\frac{p^x_{i,t-1}(M_{i,t-1}x_{i,t-1})}{P_{i,t-1}GO_{i,t-1}} = \frac{1-\gamma}{\mu_{i,t-1}}. \tag{9}$$

Finally, using (9) in (8) delivers equation (7). ■

Discussion. Two features are worth noting in proposition 1. First, Real GDP as measured by statistical agencies and Physical Value Added are identical if there is no price distortions ($\mu_{i,t-1} = 1$) and there is no love of variety in the production function ($\sigma \to \infty$). Second, the scaling term $\omega_{i,t-1}$ is the ratio of sales over nominal value added, which is a standard Domar weight. From an accounting perspective, one can decompose total sales into total cost and profits ($\Pi_{i,t}$), such that: $P_{i,t-1}GO_{i,t-1} = w_{i,t-1}L_{i,t-1} + r_{i,t-1}K_{i,t-1} + p^x_{i,t-1}(M_{i,t-1}x_{i,t-1}) + \Pi_{i,t-1}$. When total sales equal total cost, $\Pi_{i,t-1} = 0$ and the Domar weight is simply equal to the inverse of the value added share in gross output, $\omega_{i,t-1} = 1/\gamma$. In the presence of a wedge between sales and cost, we can use equations (2) and (9) to rewrite the Domar weight as:

$$\omega_{i,t-1} = \frac{P_{i,t-1}GO_{i,t-1}}{P_{i,t-1}GO_{i,t-1} - p^x_{i,t-1}(M_{i,t-1}x_{i,t-1})} = \frac{1}{1 - \frac{p^x_{i,t-1}(M_{i,t-1}x_{i,t-1})}{P_{i,t-1}GO_{i,t-1}}} = \frac{\mu_{i,t-1}}{\gamma + \mu_{i,t-1} - 1}.$$

As long as $\mu_{i,t-1}$ is close to one, the Domar weight $\omega_{i,t-1}$ is close to $1/\gamma$. However, when profits are large and $\mu_{i,t-1} > 1$, we have $\gamma\omega_{i,t-1} < 1$ and equation (7) implies that $RGDP$ reacts less than one-for-one with PVA.

Practical Implications. The difference between physical value added and measured real GDP as expressed in (7) has important implications for the interpretation of real GDP fluctuations and for our understanding of international business cycle synchronization. With $\mu_{i,t-1} > 1$ and $\sigma < \infty$, real GDP as measured in the data is not only tied to movements in technology or factor supply, but also reflects changes in the quantity and variety of imported inputs. If both $\widehat{X}_{i,t}$ and $\widehat{M}_{i,t}$ fluctuate with foreign technology shocks, equation (7) implies that an increase in the share of imported input in domestic production (i.e. a decrease in γ) raises the association between foreign shocks and domestic real GDP. International macroeconomic models that identify real GDP to physical value added cannot account for this relationship.

To better see this point, consider first a model with perfect competition and no love of variety, meaning that $\mu_{i,t-1} = 1$ and $\sigma \to \infty$. In this case, equation (7) shows that real GDP and physical value added are identical and real GDP fluctuations can only arise from changes in factor supplies and changes in technology. When simulating such a model and assuming exogenous technology shocks, a researcher would find that foreign shocks could impact domestic real GDP only to the extent that it affects factor supply. This simple observation is at the heart of the negative result presented in Kehoe and Ruhl (2008): in a model where firms take prices as given, profit maximization insures that the marginal benefit of using an additional unit of imported input is equal to its marginal cost. Hence, foreign shocks can only affect real GDP to the extent that it triggers a change in domestic factor supply. This result lies at the heart of the trade co-movement puzzle and explains why trade is not a powerful channel of propagation in standard IRBC models. In frameworks where real GDP is equal to physical value added, real GDP changes in response to a foreign shock can only arise from variations in factors supply which, in turn, are disciplined by (i) the elasticity of domestic factor supply and (ii) the complementarity between domestic factors and foreign inputs.[5] As shown in Johnson (2014), complementarity in production factors *alone* is not sufficient to solve quantitatively the TCP.

Consider now a situation where $\mu_{i,t-1} > 1$ and $\sigma < \infty$. Equation (7) reveals that changes in intermediate input usage have a first order impact on real GDP fluctuations beyond the movements of domestic factors or technology. In such a case, imported inputs yield more gains than what is reflected in their price and using more foreign inputs is associated with profits (when $\mu_{i,t-1} > 1$) or with efficiency gains (when $\sigma < \infty$).[6] All told, constructing real GDP using "base period prices" that do not reflect imported inputs' marginal revenue product creates a wedge between physical value added and real GDP fluctuations.

Importantly, the disconnect between real GDP and physical value added does not rely on the cyclicality of markups: even with constant markups, a wedge between the marginal cost and marginal revenue product of imported inputs leads to a first order impact of intermediate input usage on measured real value added. In a sense, the wedge is a purely measurement issue: when constructing real GDP, statistical agencies do not simply measure the quantity of

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[5] The role of input complementarity is discussed at length in Burstein et al. (2008) or in Boehm et al. (2019).

[6] In practice, many models feature a single parameter governing both the size of the markup and the degree of Love of Variety – this is obviously the case with CES aggregation and monopolistic competition. For clarity, equation (7) shows a specification in which the two channels are perfectly distinguishable.

goods produced, but use observed prices (fixed at a base period) to assign a value to measured quantities. If base period prices contain a markup, it creates a wedge between the marginal revenue generated by an additional unit of imported input x_i and its marginal cost p_i^x.

Finally, our results bears important implications for the calibration of macroeconomic models. As revealed by proposition 1, a researcher who builds a model with distorted prices cannot equate a model-based measure of physical value added to real GDP data in the calibration process: doing so would attribute the markup and variety effects in (7) to changes in PVA, either through technology shocks ($\widehat{Z}_{i,t}$) or through factor supply ($\widehat{L}_{i,t}$ or $\widehat{K}_{i,t}$).[7]

2.2 Productivity and Technology

The real GDP decomposition presented in equation (7) also bears important implications for the measure of productivity based on the Solow Residual (SR). As standard, we define SR so that it captures fluctuations in real GDP that are not explained by changes in domestic factors:

$$\widehat{SR}_{i,t} = \widehat{RGDP}_{i,t} - \alpha\widehat{L}_{i,t} - (1-\alpha)\widehat{K}_{i,t}\,. \tag{10}$$

Proposition 2 *When real GDP is constructed as in equation (6), the relationship between the Solow Residual SR and technology is given by:*

$$\begin{aligned} \widehat{SR}_{i,t} \;&=\; \gamma\omega_{i,t-1}\widehat{Z}_{i,t} + \underbrace{(\gamma\omega_{i,t-1}-1)\left(\alpha\widehat{L}_{i,t} + (1-\alpha)\widehat{K}_{i,t}\right)}_{\textit{Scale Effect}} \\ &\quad+\; \omega_{i,t-1}(1-\gamma)\Big(\underbrace{\frac{\mu_{i,t-1}-1}{\mu_{i,t-1}}\widehat{X}_{i,t}}_{\textit{Imported Input Effect}} + \underbrace{\frac{1}{\mu_{i,t-1}(\sigma-1)}\widehat{M}_{i,t}}_{\textit{Variety Effect}}\Big)\,. \end{aligned} \tag{11}$$

Proof: The result follows from replacing (7) in (10). ■

Discussion. When $\mu_{i,t-1} = 1$, we have $\omega_{i,t-1} = 1/\gamma$ and both the scale effect and markup effect terms in equation (11) vanish. Additionally, in absence of love of variety ($\sigma \to \infty$), the variety effect also disappears, implying that productivity is an accurate measure of technology.

In general, when $\mu_{i,t-1} > 1$ and $\sigma < \infty$, equation (11) makes it clear that productivity, as measured by the Solow Residual, is disconnected from the true technology shock $Z_{i,t}$. First,

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[7] In the present paper, we focus on the implication of precise real GDP definition for the resolution of the Trade Comovement Puzzle and hence emphasize the role of *imported* inputs. Our results can also be extended to a closed economy with multiple sectors and input-output linkages. When real GDP is computed at the sector level, proposition 1 holds and implies a disconnect between real GDP and PVA with markups and/or love of variety.

with positive markups, fluctuations in real GDP can result from movement in profits which are captured by Solow Residual fluctuations. Such profits movements can arise either from changes in domestic factors (the scale effect) or from changes in foreign input usage (the imported input effect). Second, an additional term captures the gains associated with accessing more variety from abroad whenever $\sigma < \infty$. With love of variety, this change in efficiency is also reflected in measured productivity.

The above decomposition highlights the disconnect between standard measures of productivity, such as the *Solow Residual*, and actual technology. The introduction of markups and love of variety creates new channels through which foreign shocks impact measured domestic productivity. As a result, two countries that trade intermediate inputs should have correlated Solow Residuals, a prediction we later test in the data and which our quantitative model is able to reproduce. Finally, our results can be seen as continuation of insights from Basu and Fernald (2002) or Feenstra et al. (2009), who emphasize the risk of identifying the Solow Residual to technology shocks in the calibration of a macroeconomic model.

3 A Model of International Trade with Cross-Border Input Linkages

We now put more structure on our insights and quantitatively assess the role of markups and love of variety, in conjunction with a statistically-consistent real GDP measurement, in generating a plausible TC slope. We depart from the standard IRBC model and develop a many-country international business cycle model that features trade in both final and intermediate goods, imperfect competition and extensive margin adjustments. The model is related to Ghironi and Melitz (2005) and Alessandria and Choi (2007) extended to multiple countries with homogeneous firms that are able to export and import, which implies that intermediate goods cross borders multiple times.[8]

3.1 Consumption and Labor Supply

We consider a multi-period world economy with many countries ($i, j \in \{1, ..., N\}$). Each country is populated by a representative consumer who consumes final goods $C_{i,t}$ and supplies

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[8] Alternatively, the model presented here can be thought of as an extension of the IRBC model presented in Johnson (2014) with two new elements: markups and extensive margin adjustments. It is also related to the static small open economy model in Gopinath and Neiman (2014)

labor $L_{i,t}$. Consumers' preferences are described by the following utility function:

$$U_0 = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\log\left(C_{i,t}\right) - \frac{\chi_i}{1+\nu} L_{i,t}^{1+\nu} \right] , \tag{12}$$

$$\text{with} \quad C_{i,t} = \left(\sum_{j=1}^{N} \omega_i^F(j)^{\frac{1}{\rho^F}} \cdot \widetilde{C}_{j,i,t}^{\frac{\rho^F-1}{\rho^F}} \right)^{\frac{\rho^F}{\rho^F-1}} , \quad \text{and} \quad \widetilde{C}_{j,i,t} = \left(\int\limits_{s\in\Omega^F_{j,i,t}} c_{j,i,t}(s)^{\frac{\sigma_j-1}{\sigma_j}} ds \right)^{\frac{\sigma_i}{\sigma_i-1}} , \tag{13}$$

where χ_i is a labor disutility scaling parameter, β is the rate of time preference, ν the inverse of the Frisch elasticity of labor supply and σ_i the elasticity of substitution between different varieties of final goods $c_{i,j,t}(s)$ produced by firm s originating from country i and serving country j. $\omega_i^F(j)$ measures the share of country j's final good $\widetilde{C}_{j,i,t}$ in the *consumption* bundle of country i, with $\sum\limits_{j=1}^{N} \omega_i^F(j) = 1$, and $\Omega^F_{j,i,t}$ is the endogenous set of firms from country j that serve the *final good* market in country i. Finally, ρ^F is the *final goods* Armington elasticity of substitution. Final good price indexes are defined as:

$$\mathcal{P}^F_{i,t} \; = \left(\sum_{j=1}^{N} \omega_i^F(j) \cdot \left(\widetilde{\mathcal{P}}^F_{j,i,t}\right)^{1-\rho^F} \right)^{\frac{1}{1-\rho^F}} , \quad \text{and} \quad \widetilde{\mathcal{P}}^F_{j,i,t} = \left(\int\limits_{s\in\Omega^F_{j,i,t}} p^F_{j,i,t}(s)^{1-\sigma_i} ds \right)^{\frac{1}{1-\sigma_i}} , \tag{14}$$

where $p^F_{j,i,t}(s)$ is the price charged by firm s in the set $\Omega^F_{j,i,t}$ when selling in the *final good* market in country i. As we will see below, given our assumptions, firms charge the same price in both final and intermediate good markets in a given country.

Asset Markets. Our benchmark economy assumes financial autarky between countries, so that agents choose consumption $C_{i,t}$, investment $\mathcal{I}_{i,t}$, and labor $L_{i,t}$, subject to:[9]

$$\mathcal{P}^F_{i,t} \left(C_{i,t} + \mathcal{I}_{i,t}\right) = w_{i,t} L_{i,t} + r_{i,t} K_{i,t} - \mathcal{T}_{i,t} \,, \tag{15}$$

$$K_{i,t+1} = (1-\delta) K_{i,t} + \mathcal{I}_{i,t} - \frac{\psi}{2} \left(\frac{\mathcal{I}_{i,t}}{K_{i,t-1}} - \delta \right)^2 K_{i,t} \,, \tag{16}$$

where the term $\mathcal{T}_{i,t}$ captures potential trade imbalance in country i, i.e. $\mathcal{T}_{i,t} < 0$, corresponds to a trade deficit meaning that country i consumes more than the value of its production. Capital $K_{i,t}$ depreciates at the rate δ and investment entails quadratic adjustment costs governed by

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[9] Note that the right hand side of (15) includes firms' profits since, as explained below, firms pay entry costs using domestic labor. It should then be understood that $L_{i,t}$ includes both production and "entry cost" workers.

the parameter ψ. Financial autarky is our preferred benchmark as it yields a closer fit to some key business cycle moments relative to a version under complete markets (Heathcote and Perri, 2002). In section 5.3, we show that the financial autarky assumption is not critical for the results as complete financial markets deliver similar results regarding the TC slope.

Given prices, consumers choose $\{C_{i,t}, L_{i,t}, K_{i,t+1}\}$ to maximize (12) subject to (15) and (16).

3.2 Production Side

In country i, production is performed by a continuum of homogeneous firms with productivity $Z_{i,t}$. Firms produce with a Cobb-Douglas technology using labor $\ell_{i,t}$, capital $k_{i,t}$, and intermediate inputs $X_{i,t}$ bought from both home and foreign firms. The intermediate input index, $X_{i,t}$, is a CES aggregation of country-pair specific bundles $\widetilde{X}_{j,i,t}$, with an *intermediate goods* Armington elasticity ρ^I. To introduce a rationale for markups and for love of variety, each country-pair specific bundle is itself a CES aggregation of many varieties, with an elasticity of substitution σ_j. Production technology for a firm in i writes:

$$q_{i,t} = \left(Z_{i,t}\ell_{i,t}^{\alpha}k_{i,t}^{1-\alpha}\right)^{\gamma_i} X_{i,t}^{1-\gamma_i}\,, \tag{17}$$

$$\text{with} \quad X_{i,t} = \left(\sum_{j=1}^{N} \omega_i^I(j)^{\frac{1}{\rho^I}}\, \widetilde{X}_{j,i,t}^{\frac{\rho^I-1}{\rho^I}}\right)^{\frac{\rho^I}{\rho^I-1}}, \quad \text{and} \quad \widetilde{X}_{j,i,t} = \left(\int\limits_{s\in\Omega^I_{j,i,t}} x_{j,i,t}(s)^{\frac{\sigma_i-1}{\sigma_i}}\, ds\right)^{\frac{\sigma_i}{\sigma_i-1}}, \tag{18}$$

where γ_i is the share of value added in gross output, $\omega_i^I(j)$ measures the share of country j's intermediate inputs $\widetilde{X}_{j,i,t}$ in the *production process* of country i, with $\sum_{j=1}^{N}\omega_i^I(j) = 1$, and $\Omega^I_{j,i,t}$ is the endogenous set of firms producing intermediate input $x_{j,i,t}$ based in j and serving the *intermediate input* market in country i. Similarly to the *final good* market, we have

$$\mathcal{P}^I_{i,t} = \left(\sum_{j=1}^{N} \omega_i^I(j)\left(\widetilde{\mathcal{P}}^I_{j,i,t}\right)^{1-\rho^I}\right)^{\frac{1}{1-\rho^I}}, \quad \text{and} \quad \widetilde{\mathcal{P}}^I_{j,i,t} = \left(\int\limits_{s\in\Omega^I_{j,i,t}} p^I_{j,i,t}(s)^{1-\sigma_i} ds\right)^{\frac{1}{1-\sigma_i}}, \tag{19}$$

$$\text{and } \mathcal{P}^{IB}_{i,t} = \left(\frac{w_{i,t}}{\alpha\gamma_i}\right)^{\alpha\gamma_i}\left(\frac{r_{i,t}}{(1-\alpha)\gamma_i}\right)^{(1-\alpha)\gamma_i}\left(\frac{\mathcal{P}^I_{i,t}}{1-\gamma_i}\right)^{1-\gamma_i}, \tag{20}$$

where $\widetilde{\mathcal{P}}^I_{j,i,t}$ denotes the price of the country-pair specific bundle $\widetilde{X}_{j,i,t}$ and $\mathcal{P}^{IB}_{i,t}$ is the unit cost of the Cobb Douglas bundle aggregating $X_{i,t}$, $k_{i,t}$, and $\ell_{i,t}$ (called the *input bundle*) and represents the price of the basic production factor in country i. $p^I_{j,i,t}(s)$ is the price charged by any firm s

in the set $\Omega^I_{j,i,t}$ when selling in the *intermediate input* market in country i.

Finally, there is an overhead entry cost f^E_i, sunk at the production stage. Based on their expected profit in all markets, firms enter the economy until the expected value of doing so equals the overhead entry cost. This process determines the mass of firms $M_{i,t}$. Absent firms heterogeneity, as in Krugman (1980), all firms export to all markets which implies that $\Omega^F_{j,i,t} \equiv \Omega^I_{j,i,t} \equiv [0, M_{i,t}]$.[10] The free entry condition in country i can be written as:

$$\Pi_{i,t} = M_{i,t} \frac{w_{i,t}}{Z^{\gamma_i}_{i,t}} . f^E_i \, , \tag{21}$$

where the sunk cost f^E_i is labeled in labor units and $\Pi_{i,t}$ is aggregate profits in country i.

3.3 Equilibrium

Let us define $Y_{i,t}$ as the aggregate income of consumers in country i, and $S_{i,t}$ as the total spending of firms in the same country (both in nominal terms). Given prices, total demand faced by a firm in country i is the sum of demand stemming from *final good* and *intermediate input* markets in all countries:

$$q_{i,t} = \sum_{j=1}^{N} \Bigg[\underbrace{\left(\frac{p^F_{i,j,t}}{\widetilde{\mathcal{P}}^F_{i,j,t}}\right)^{-\sigma_i} \left(\frac{\widetilde{\mathcal{P}}^F_{i,j,t}}{\mathcal{P}^F_{j,t}}\right)^{-\rho^F} \frac{\omega^F_j(i) Y_{j,t}}{\mathcal{P}^F_{j,t}}}_{\text{Final goods demand}} + \underbrace{\left(\frac{p^I_{i,j,t}}{\widetilde{\mathcal{P}}^I_{i,j,t}}\right)^{-\sigma_i} \left(\frac{\widetilde{\mathcal{P}}^I_{i,j,t}}{\mathcal{P}^I_{j,t}}\right)^{-\rho^I} \frac{\omega^I_j(i)(1-\gamma_j) S_{j,t}}{\mathcal{P}^I_{j,t}}}_{\text{Intermediate goods demand}} \Bigg], \tag{22}$$

where the summation is done over all markets that are served by a firm. To be allowed to sell its variety to a country j, firms from country i must pay a variable (iceberg) cost τ_{ij}. Given this cost, they choose their prices to maximize profits. With constant price elasticity of demand, they charge a constant markup over marginal cost. For a firm from country i, the only elasticity that is relevant for pricing is σ_i, capturing the fact that their individual pricing decision has no impact on country-specific price indexes. As a result, firms charge the same markup in the final and intermediate good markets, and we have: $p^F_{i,j,t} = p^I_{i,j,t} = p_{i,j,t}$ and $\widetilde{\mathcal{P}}^F_{i,j,t} = \widetilde{\mathcal{P}}^I_{i,j,t} = \widetilde{\mathcal{P}}_{i,j,t}$. The marginal cost of a firm in country i is $\mathcal{P}^{IB}_{i,t} / (Z^{\gamma_i}_{i,t})$ and its optimal price in country j is:

$$p_{i,j,t} = \tau_{ij} \frac{\sigma_i}{\sigma_i - 1} \frac{\mathcal{P}^{IB}_{i,t}}{Z^{\gamma_i}_{i,t}} \, . \tag{23}$$

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[10] In an earlier version of this paper, we also introduced heterogeneity with firm's idiosyncratic productivity as in Ghironi and Melitz (2005) or Fattal Jaef and Lopez (2014). This feature generates different masses of firms supplying final goods and intermediate inputs in each sub-market (i, j). The results are quantitatively similar to those obtained in a simplified version with homogeneous firms, and we therefore dropped this layer of complexity in the baseline.

Unlike Krugman (1980) or Ghironi and Melitz (2005), one needs to jointly solve for all prices in the economy. Through $\mathcal{P}^{IB}_{i,t}$, the price charged by a firm in country i depends on the prices charged by all firms supplying country i, which in turn depend on the prices charged by their suppliers and so on and so forth. Determining prices requires solving jointly for all country-pair specific price indexes in every market. Observing that $\widetilde{\mathcal{P}}_{i,j,t} = \tau_{ij}\widetilde{\mathcal{P}}_{i,i,t}$, the definition of price indexes in every country yields a system of N equations which jointly defines all *inner* price indexes:

$$\widetilde{\mathcal{P}}_{i,i,t} = \xi_i \left(\sum_{j=1}^{N} \omega^I_i(j) \left(\tau_{ji} \widetilde{\mathcal{P}}_{j,j,t} \right)^{1-\rho^I} \right)^{\frac{1-\gamma_i}{1-\rho^I}} , \tag{24}$$

with ξ_i depending on the mass of firms, the price of labor and capital, and parameters.[11] For given mass of firms, this system admits a unique non-negative solution.[12]

Closing the model involves standard market clearing conditions for capital, labor, and goods. Total revenues $R_{i,t}$ of all firms from country i can be written as:

$$R_{i,t} = \sum_{j=1}^{N} \left[\left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}^F_{j,t}} \right)^{1-\rho^F} \omega^F_j(i) Y_{j,t} + \left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}^I_{j,t}} \right)^{1-\rho^I} \omega^I_j(i)(1-\gamma_j) S_{j,t} \right] . \tag{25}$$

Total exports from i to j, $T_{i\to j,t}$, are the sum of final goods and intermediate inputs exports, defined as:

$$T_{i\to j,t} = \left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}^F_{j,t}} \right)^{1-\rho^F} \omega^F_j(i) Y_{j,t} + \left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}^I_{j,t}} \right)^{1-\rho^I} \omega^I_j(i)(1-\gamma_j) S_{j,t} \, . \tag{26}$$

Consumer's revenues $Y_{i,t}$ are equal to the sum of the payment to production workers $\alpha\gamma_i S_{i,t}$, rent from capital $(1-\alpha)\gamma_i S_{i,t}$, total firms' profits $\Pi_{i,t}$ (which, at the free entry equilibrium, is completely used to pay the entry cost f^E_i), and potential trade imbalances $-\mathcal{T}_{i,t}$. Moreover, in absence of fixed cost of exporting, note that both profits and spending can be expressed as a function of revenues with $\Pi_{i,t} = \frac{1}{\sigma_i} R_{i,t}$ and $S_{i,t} = \frac{\sigma_i - 1}{\sigma_i} R_{i,t}$. Using $Y_{i,t} = w_{i,t}L_{i,t} + r_{i,t}K_{i,t} - \mathcal{T}_{i,t} = \gamma_i S_{i,t} + \Pi_{i,t} - \mathcal{T}_{i,t}$, equation (25) can be written in compact form as:

$$\mathbf{G}_t \cdot \mathbf{R}_t{}' = -\left[\mathbf{W}^{F\prime}_t \circ \mathbf{P}^F_t \right] \boldsymbol{\mathcal{T}}_t{}' , \tag{27}$$

where $\boldsymbol{\mathcal{T}}_t$ and $\mathbf{R}_t$ are vectors that stack trade imbalances and total revenues of all firms. $\mathbf{W}^F_t$ is

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[11] From equations (19), (20) and (23), we have: $\xi_i = M_{i,t}^{\frac{1}{1-\sigma_i}} \left(\frac{\sigma_i}{\sigma_i - 1} \left(\frac{w_{i,t}}{\alpha\gamma_i} \right)^{\alpha\gamma_i} \left(\frac{r_{i,t}}{(1-\alpha)\gamma_i} \right)^{(1-\alpha)\gamma_i} \left(\frac{1}{1-\gamma_i} \right)^{1-\gamma_i} \frac{1}{Z_{i,t}^{\gamma_i}} \right)$.

[12] Following Kennan (2001) and denoting $B_k = (\widetilde{\mathcal{P}}_{i,i,t})^{1-\rho^I}$ and $\mathbf{B}$ the associated $N \times 1$ vector, it suffices to show that the system is of the form $\mathbf{B} = f(\mathbf{B})$ with $f : \mathbb{R}^N \to \mathbb{R}^N$ a vector function which is strictly concave with respect to each argument, which is obvious as long as $0 < \gamma_i < 1$ for all countries.

the weighting matrix associated with final good aggregation and whose elements are defined as $W^F_{ij} = \omega^F_i(j)$, $\mathbf{P}^F_t$ is a matrix defined by elements $P^F_{i,j,t} = \left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}^F_{i,t}}\right)^{1-\rho^F}$, and $\circ$ is the element-wise (Hadamard) product. The matrix $\mathbf{G}_t$ is defined as:

$$G_{i,j,t} = \mathbb{1}_{\{j=i\}} - \left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}^F_{j,t}}\right)^{1-\rho^F} \omega^F_j(i)\frac{\gamma_j(\sigma_j-1)+1}{\sigma_j} - \left(\frac{\widetilde{\mathcal{P}}_{i,j,t}}{\mathcal{P}^I_{j,t}}\right)^{1-\rho^I} \omega^I_j(i)(1-\gamma_j)\left(\frac{\sigma_j-1}{\sigma_j}\right). \quad (28)$$

Finally, labor is used for production ($L^p_{i,t}$) or for the entry cost ($L^e_{i,t}$) so that $L_{i,t} = L^p_{i,t} + L^e_{i,t}$, and firms' symmetry implies that $M_{i,t}\ell_{i,t} = L^p_{i,t}$. Setting $w_{1,t} = 1$, such that $S_{1,t} = L^p_{1,t}/(\alpha_1\gamma_1)$, provides a unique solution for all variables by solving together the consumer problem (12), the price system (24), the free entry system (21), and the revenue system (27).

3.4 Real Value Added definitions

As in section 2, we introduce two measures of value added: a *model-based* measure of physical value added (PVA) and a *statistical* measure of real GDP ($RGDP$). Only the latter index is comparable to the data produced by statistical agencies.

Physical Value Added. Thanks to separability between domestic factor and inputs in the firm-level production function (17), aggregating physical value added across all firms yields $PVA_{i,t} = Z_{i,t}L^{\alpha}_{i,t}K^{1-\alpha}_{i,t}$. This measure of real value added is unit-less and, following Arrow (1974), one can interpret it as a measure of a purely theoretical bundle that is used, in combination with intermediate inputs, to produce the gross output.

Real GDP. Real GDP as constructed by statistical agencies is not unit-less but uses prices at their base period level to express each component in a commonly accepted unit of account.[13]

In most databases, real GDP is defined using the Fisher ideal quantity index which is a geometric mean of the Laspeyres and Paasche indices. Hence, for any period t, the base period price used in the construction of real GDP growth from $t-1$ to t is a geometric mean between period t and period $t-1$ prices. To be as close as possible to the method used in the construction of the data while simplifying the analysis, we define real GDP (RGDP) using steady state prices as base-period prices.[14] Real GDP is obtained by deflating nominal spending using price indexes that are corrected from product variety effects to measure "quantity indices", and then

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[13] In most cases, real GDP is constructed using chain weighted prices, as discussed here. Some database report real GDP in "constant prices" where prices used in the construction of real value added are fixed at a reference year. Obviously, no database reports real GDP by "counting the number of goods" produced in a country.

[14] In section 5.3, we examine advanced base-period prices utilized by the Bureau of Economic Analysis.

by valuing these "quantity indices" using steady-state prices. More precisely:

$$RGDP_{i,t} = \underbrace{\underbrace{\widehat{\mathcal{P}_i^{F,ss}} \; \frac{Y_{i,t}}{\widehat{\mathcal{P}_{i,t}^F}}}_{\text{Consumption + Investment}} + \underbrace{\sum_{j=1,j\neq i}^{N} \widehat{\mathcal{P}_{i,j}^{ss}} \; \frac{T_{i\to j,t}}{\widehat{\mathcal{P}_{i,j,t}}}}_{\text{Total exports (final+ inputs)}}}_{\text{= Gross Output + Imported Final Good}} - \underbrace{\sum_{j=1,j\neq i}^{N} \widehat{\mathcal{P}_{j,i}^{ss}} \; \frac{T_{j\to i,t}}{\widehat{\mathcal{P}_{j,i,t}}}}_{\text{Total imports (final + inputs)}} \tag{29}$$

where, in order to be consistent with the way actual data are collected, we defined variety-corrected price indexes as $\widehat{\mathcal{P}_{i,j,t}} = (M_{i,t})^{1/(\sigma_i-1)} \, \widetilde{\mathcal{P}}_{i,j,t}$ and $\widehat{\mathcal{P}_{i,t}^F} = \left(\sum_{j=1}^{N} \omega_i^F(j) \cdot \left(\widehat{\mathcal{P}_{j,i,t}}\right)^{1-\rho^F}\right)^{\frac{1}{1-\rho^F}}$. Since both consumers' utility and production functions have a CES component, it is well known that the associated price indexes can be decomposed into components reflecting average prices (captured by statistical agencies) and product variety (which is not taken into account in national statistics).[15]

In equation (29), we defined RGDP from the expenditure side. One could also define RGDP from the production side, by summing gross domestic output sold in all markets and subtracting imported inputs. All our results are unchanged using such alternative measure.

3.5 Calibration of the Model Parameters

We solve the model with 35 countries and a composite rest-of-the-world for the period spanning from 1970 to 2009. Those countries represent around 91% of world GDP, 64% of total trade flows, 56% of total trade in final goods and 72% of total trade flows in intermediate inputs.[16] Within those countries, the share of trade in intermediate inputs in total trade flows is about 57%. The model's period is set to the quarter.

Parameters. Table 1 reports fixed parameters. Starting with parameters that are identical across countries, we set $\beta = 0.995$, $\delta = 0.025$, and $\nu = 0.5$. These values imply a Frisch elasticity of 2 and an annual real interest rate of 12%, which is slightly higher than the average of 9% in the Penn World Table 10.0 (PWT) for the considered countries. We set $\alpha = 0.6$, which corresponds to the average labor share in the World Input-Output Database (WIOD) and the PWT for our sample of countries and time coverage. The macro (Armington) elasticities, ρ^I and

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[15] See for example the illuminating discussion in Feenstra (1994) or Ghironi and Melitz (2005). In section 5.3, we show that the results without this correction leads to an over-estimation of the TC slope.

[16] The sample of countries, including Argentina, Australia, Austria, Belgium, Brazil, Canada, Switzerland, Chile, China, Denmark, Spain, Finland, France, Germany, Greece, Indonesia, India, Ireland, Israel, Italy, Japan, South Korea, Mexico, Netherlands, Norway, New Zealand, Poland, Portugal, Sweden, Thailand, Turkey, United Kingdom, United States, Vietnam, and South Africa, is chosen to obtain a balanced panel data of bilateral trade flows, using data from Johnson (2014).

ρ^F, are set to 1, in line with literature.[17] There is also a theoretical convenience for this choice, as it allows the model to take the same form as other network models such as Acemoglu et al. (2012). The degree of capital adjustment costs, ψ, is chosen to obtain a volatility of investment with respect to real GDP consistent with the data.[18]

We then proceed to country-specific parameters. The parameter χ_i is chosen to replicate the *relative difference* of working age population. We set a value of $\sigma_i = \sigma = 4.0$, $\forall i$ for the micro elasticity of substitution, in line with the average markups reported in De Loecker and Eeckhout (2018) for our sample of countries.[19] Anderson and van Wincoop (2004) reports estimates in the range of 3 to 10. Following Bernard et al. (2003), Ghironi and Melitz (2005) choose a micro elasticity of 3.8 and recently, papers such as Barrot and Sauvagnat (2016) or Boehm et al. (2019) argue that firms' ability to substitute between their suppliers can be very low. This choice leads to markups of around 33%. As a robustness, we also consider alternative elasticities for σ_i in section 5.1.2, and defer discussion of those cases till then.

Table. 1. Benchmark Parameter Values.

Parameter	**Symbol**	**Value**	**Moment / Source**
Time preference rate	β	.995	Annual real interest rate of 12%
Labor curvature	ν	.50	Frisch elasticity of 2.0
Labor disutility	χ_i	[.00, .2]	Relative working age population
Labor exponent	α	.60	Mean labor share, Penn World Tables
Intermediate exponent	γ_i	[.10, .44]	Share of intermediate cost in total sales, WIOD
Investment adj. cost	ψ	5.00	Mean investment volatility $\frac{std(I)}{std(RGDP)}$ between 3.0-4.5
Depreciation rate	δ	.025	10% annual depreciation
Argminton elasticities	ρ^I, ρ^F	1.00	Saito (2004), Feenstra et al. (2014)
Elasticity of substitution	$\sigma_i, \forall i$	4.00	33% Markup, De Loecker and Eeckhout (2018)
Sunk entry cost	f_i^E / f_{US}^E	[.1, 14.9]	*Doing Business Database* - World Bank
Iceberg trade cost	τ_{ij}	[1 - 3.6]	ESCAP - World Bank

The value added shares, γ_i, are calibrated using data on cost of intermediates and total sales from the 2000-2009 years at the 2-digit sector level in the WIOD database. Specifically, $(1 - \gamma_{i,s})$ represents the share of intermediate inputs in total costs in a given sector s. Following Basu (1995), we use the fact that $total\ sales_s = \mu_i \times total\ cost_s$, with μ_i the markups in country i. As such, the intermediate inputs exponent is $\gamma_{i,s} = 1 - \frac{cost\_intermediates_s}{total\_sales_s} \frac{\sigma_i}{\sigma_i - 1}$. The implied mean

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[17] For comparison, Saito (2004) provides estimations from 0.24 to 3.5 for the Armington elasticity. Studying macro and micro elasticities for final goods, Feenstra et al. (2014) finds that, for the majority of goods, the macro elasticity is lower than the micro elasticity.

[18] The standard business cycle statistics of our model can be found in the online appendix, section OA3.3.

[19] They use a cost-based approach to measure aggregate markups, which they define as the ratio of the output price to the marginal costs, and therefore relies solely on information from the financial statements of firms (sales value and cost of goods sold). Aggregating all firms specific markups for each country, they provide a detailed and comparable measure of market power for 29 of our countries from 1980 to 2016.

values of γ_i, weighted by the sector importance in total sales, range from 0.10 to 0.44 for our sample, in line with values reported in Halpern et al. (2015), and Gandhi et al. (2020).[20]

To measure relative entry fixed costs, f_i^E / f_{US}^E, we use the time required to establish a business in each country relative to the US from the *Doing Business Indicators*. We normalize f_{US}^E to generate a ratio of the total number of firms to the working population, $\frac{M}{L}$, of approximately 12%.[21] To measure variable (iceberg) trade costs, τ_{ij}, we use the ESCAP World Bank's International Trade Costs Database, where we normalize $\tau_{ii} = 1$. This database includes symmetric bilateral trade costs in the broader sense, such as transport costs, tariffs and other components discussed in Anderson and van Wincoop (2004).

Steady-state trade shares. Data on import shares, $\left\{ \frac{T^I_{j \to i}}{RGDP_i}, \frac{T^F_{j \to i}}{RGDP_i} \right\}$, are sufficient to identify the share of foreign goods in final demand and input use, $\omega_i^I(j)$ and $\omega_i^F(j)$. To evaluate the TC slope and be as close as possible to the empirical analysis which is presented below, we successively calibrate our model to four different time windows. Using trade data separated into final and intermediate goods from Johnson and Noguera (2017), and GDP data from the PWT, we calibrate the trade shares using the average values for 1970-1979, 1980-1989, 1990-1999, and 2000-2009 successively.[22]

Since complete financial autarky is inconsistent with the trade imbalances observed in the data, we calibrate the model's trade imbalances $\{\mathcal{T}_1, ..., \mathcal{T}_N\}$ to match those relative to real GDP in each time window, and maintain those nominal imbalances constant during the simulation. In this way, the model's steady state precisely replicates relative bilateral trade flows and trade imbalances.

Aggregate Technology Process. The level of comovement of real GDP among countries in our simulations is influenced by both the correlation of exogenous technology shocks and the internal transmission of those shocks among countries. We model the country-specific process for technology $Z_{i,t}$ using an autoregressive (AR) process of order 1, such that: $\log(Z_{i,t}) = \rho_Z \log(Z_{i,t-1}) + \epsilon^Z_{i,t}$, where $\epsilon^Z_{i,t}$ is a shock with mean zero and covariance-variance matrix Σ.

As discussed in section 2, TFP measures based on the Solow Residual are a biased measure of technology, as they are affected by the extent to which countries use imported intermediate

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[20] There are seven countries that are absent in the WIOD (2000-2009): Argentina, Chile, Isreal, New Zealand, Thailand, Vietnam and South Africa. We assign the mean value of γ_i for those countries and the *RoW*.

[21] There is about 22-24 millions of non-employer businesses and 5.5 millions of employer businesses in the US, while the working age population represents around 180 millions of individuals during the considered period. Consistently, the self-employment rate is around 12% for the US between 1990 and 2000 (BLS). Results are not sensitive to this assumption.

[22] We provide additional details on data sources in the online appendix OA1.1.

inputs. However, such measures are still the best available proxy for $Z_{i,t}$. To calibrate the off-diagonal elements of the covariance matrix Σ, we use the real TFP variable in the PWT at annual frequency (detrended using HP-filter with smoothing value of 6.25), denoted $\widehat{\Sigma}$. Additionally, to make sure our simulations deliver a median correlation of real GDP in line with the data, we use a scaling factor v such that $\Sigma = v\widehat{\Sigma}$, where v is selected to ensure that the median RGDP correlation in our simulation is equal to the correlation observed in the data, of about 0.32.[23] In this procedure, we set the cross-country TFP correlation of each country with the composite rest-of-the-world to its observed average value. In section 5.3, we present results under uncorrelated shocks, negatively correlated shocks, and other variations.

To generate realistic real GDP fluctuations in the simulated economy, we set the variance of TFP shocks (σ^Z) and the AR(1) persistence (ρ^Z) to match a de-trended and HP-filtered real GDP volatilities of 2% and an average auto-correlation of 0.75, in line with estimates from Aguiar and Gopinath (2007) with a sample of developing and developed countries.[24] Table 2 reports parameters that are calibrated to match the above empirical moments.

Table. 2. Calibrated parameters of the model.

	Parameter	**Value**	**Main target**
Inputs spending weights	$\omega_i^I(j)$	*in text*	Import shares in inputs
Final goods spending weights	$\omega_i^F(j)$	*in text*	Import shares in final goods
Trade imbalance	$\{\mathcal{T}_i, ..., \mathcal{T}_N\}$	*in text*	Trade imbalance over GDP
Persistency of TFP shocks	ρ_Z	.70	Avg. RGDP auto-correlation of 0.75
Std. of TFP shocks	σ^Z	.25%	Avg. RGDP volatility of 2%
Scaling of TFP covariance	v	.45	Median cross-country RGDP correlation of 0.32

To get a sense of the international propagation property of our model, note that in order to match a median international real GDP correlation ($RGDP$) of 0.32, the median correlation of technology shocks is only 0.09. For reference, the median correlation of physical value added (PVA) in such a simulation is only 0.15, much lower than $RGDP$. Interestingly, when we set all trade linkages to zero and keep the same structure of technology shocks, the median correlation of both $RGDP$ and PVA is 0.13, highlighting that trade linkages play an important role in the synchronization of $RGDP$, while it plays a limited role for the synchronization of PVA.

The remainder of the paper investigates the channels through which trade linkages transmit international shocks and synchronize business cycles across countries, focusing more precisely on the TC slope.

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[23] Importantly, recall that the goal of our exercise is not to explain the ***level*** of comovement across countries, but its ***slope*** following a change in trade intensities.

[24] The results are not affected by assuming heterogeneity in the standard deviations across countries.

4 Results

We now quantitatively gauge how our model can contribute to explain the trade-comovement puzzle. We begin by providing an overview of the empirical relationship between trade intensity and comovement. We then evaluate the model's ability to account for these findings by focusing on two questions. How well can our model replicate the observed TC slope? What role do markups, extensive margin adjustments, and real GDP measurement play in generating the TC slope?

4.1 The Trade-Comovement Slope: Data

We refine the seminal Frankel and Rose (1998) analysis on the relationship between bilateral trade intensity and GDP comovement, and depart from their original study in two ways. First, we break down total trade into trade in intermediate inputs and trade in final goods. This distinction is important because our theory in section 2 predicts that domestic RGDP reacts specifically to intermediate inputs trade. Second, we make use of a panel estimation and exploit variations within country-pairs to estimate the relationship between changes in bilateral trade linkages and changes in GDP comovement.

Our panel consists of the same 35 countries from the model, covering the time period of 1970 to 2009. To construct cross-country RGDP correlation, we use real GDP measured at chained PPPs from the PWT, which is detrended in two ways: (i) HP filter with smoothing parameter 6.25 to capture the business cycle frequencies and (ii) log first difference. Similar to our model calibration, bilateral trade flows for final goods and trade in intermediate inputs are taken from Johnson and Noguera (2017). As standard in the literature, we assess the role of trade proximity on GDP comovement by creating symmetric measures of bilateral trade intensity using the sum of total exports ($T^d_{i\to j,t}$) and total imports ($T^d_{j\to i,t}$) from country i to j in category $d \in \{\text{input}, \text{final}\}$, divided by the country-pair GDPs, as follows: $\text{Trade}^d_{ij,t} = \frac{T^d_{i\to j,t} + T^d_{j\to i,t}}{GDP_{i,t} + GDP_{j,t}}$.

The extent to which countries have correlated GDP can be influenced by many factors beyond international trade, including correlated shocks, financial linkages, common monetary policies, etc. Because those other factors can themselves be correlated with the index of trade proximity in the cross section, using cross-section identification could yield biased results.[25] To separate the effect of trade linkages from other unobservable elements, we construct a panel dataset by creating four time-window of ten years each, indexed b, such that $b \in \{$1970-

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[25] This limitation of cross sectional analysis has also been discussed by Imbs (2004), who notes that bilateral trade intensity can be a proxy of country-pair similarity, and thus of correlated shocks.

1979, ..., 2000-2009}. Within each time window, we compute GDP correlation (Corr GDP) as well as the average bilateral trade intensities defined above.

Our empirical strategy relies on the estimation of the following specifications:

$$\text{Corr GDP}_{ijb} = \beta_1 \ln(\text{Trade}^{\text{input}}_{ijb}) + \beta_2 \ln(\text{Trade}^{\text{final}}_{ijb}) + \mathbf{X}_{ijb} + \text{CP}_{ij} + \text{TW}_b + \epsilon_{ijb} \tag{30}$$

where i and j denote the two countries within a given country-pair ij. CP_{ij} and TW_b stand for country-pair and time windows fixed effects. The set of controls $\mathbf{X}_{ijb}$ include dummy variables for countries among trade union: the different waves of the European Unions, the Euro Area, and the USSR. We later include a set of additional controls that we discuss below.

Table 3 shows that when using within country-pair variations, trade in intermediate inputs is significantly and positively associated with higher GDP co-movement (columns (1) and (4)). This result confirms the cross-sectional estimates in Di Giovanni and Levchenko (2010), who investigate the role of vertical linkages in output synchronization using I/O matrices from the BEA. Columns (2) and (5) reveal that the relationship between trade in intermediate inputs and GDP co-movement is robust to the inclusion of time-windows fixed effect. Our estimates are also economically significant. Based on estimates in column (2) and noting that the log trade intensity in intermediate goods between the time periods 1970-1979 and 2000-2009 has doubled, the slope coefficient implies an associated surge of GDP correlation of 14.5%, a non negligible increase. In contrast, trade in final goods is insignificant, or weakly negatively correlated.

Table. 3. Panel estimation: Trade proximity and GDP correlation.[a]

	Corr GDP$^{\text{HP filter}}$			Corr ΔGDP		
	(1)	(2)	(3)	(4)	(5)	(6)
$\ln(\text{Trade}^{\text{input}})$	0.067^{***}	0.063^{**}	0.065^{**}	0.072^{***}	0.050^{**}	0.049^{**}
	(0.026)	(0.025)	(0.026)	(0.024)	(0.024)	(0.024)
$\ln(\text{Trade}^{\text{final}})$	-0.000	-0.023	-0.033	-0.032	-0.013	-0.022
	(0.024)	(0.026)	(0.026)	(0.022)	(0.025)	(0.025)
Country-pair fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Time windows fixed effects	No	Yes	Yes	No	Yes	Yes
Additional controls	No	No	Yes	No	No	Yes
N	2,380	2,380	2,380	2,380	2,380	2,380
Within R^2	0.085	0.165	0.167	0.076	0.139	0.146

Notes: *p<0.1; **p<0.05; ***p<0.01. SE clustered on country-pairs.

Robustness of the Empirical Slope. Our results are robust to additional controls that capture the similarity of *trade networks*, which measures common exposure to third countries, and *sectoral composition* of trade. Our "network proximity" index is motivated by the fact that two countries with similar trade partners could co-move because of their common exposure to other countries. We define $network_{ijb}^{prox} = 1 - \frac{1}{2} \sum_{k \neq i,j} \left| \frac{T_{i \to k,b}^{\text{total}} + T_{k \to i,b}^{\text{total}}}{\sum_s T_{i \to s,b}^{\text{total}} + T_{s \to i,b}^{\text{total}}} - \frac{T_{j \to k,b}^{\text{total}} + T_{k \to j,b}^{\text{total}}}{\sum_s T_{j \to s,b}^{\text{total}} + T_{s \to j,b}^{\text{total}}} \right|$. This metric quantifies the degree of similarity in the geographical distribution of trade shares between country i and country j. It is equal to 0 if countries i and j have completely different trade partners, and it is equal to 1 if their trade shares are identical.[26] The "sectoral proximity" index is defined as $sector_{ijb}^{prox} = 1 - \frac{1}{2} \sum_{s \in \mathcal{S}} \left| \frac{T_{i,b}(s)}{\sum_{s \in \mathcal{S}} T_{i,b}(s)} - \frac{T_{j,b}(s)}{\sum_{s \in \mathcal{S}} T_{j,b}(s)} \right|$ with $T_{i,b}(s)$ the total export of country i in the specific sector s in the set of all sectors $\mathcal{S}$, and controls for changes in specialization. If two countries export exactly the same share of each product, then the index is equal to 1. If shocks have a sectoral component, then two countries that tend to specialize over time in the same sectors could have an increase in business cycle comovements over and beyond any trade effects. For those two indexes, we use bilateral trade data (SITC4 REV. 2) from the Observatory of Economic Complexity. As shown in Table 3, the results are robust to the inclusion of sector proximity and network proximity (columns (3) and (6)). Finally, we test a wide range of alternative specifications and filtering methods, different sample selection varying country and time coverage, variable definitions as well as additional controls in section OA1.3 of the online appendix. In all our specifications, trade in intermediate inputs is significantly and positively associated with higher cross-country GDP correlation, a finding consistent with our measurement theory.

4.2 Quantifying the Trade Comovement Puzzle: Model vs. Data

To assess the model's ability to replicate the link between trade and real GDP-comovement, we compute the pairwise correlation of logged and HP-filtered $RGDP$ and PVA in the model with correlated TFP shocks. We successively calibrate our model to the same four time windows of 10 years used in the empirical analysis, successively matching the level of final good and intermediate input import shares $\{\omega_i^F(j), \omega_i^I(j)\}$ observed in the data. For each time window thus calibrated, we perform 100 simulations of 10-year periods each, and record the average correlation of $RGDP$ and PVA.[27] For reference, bilateral trade intensities increased significantly over

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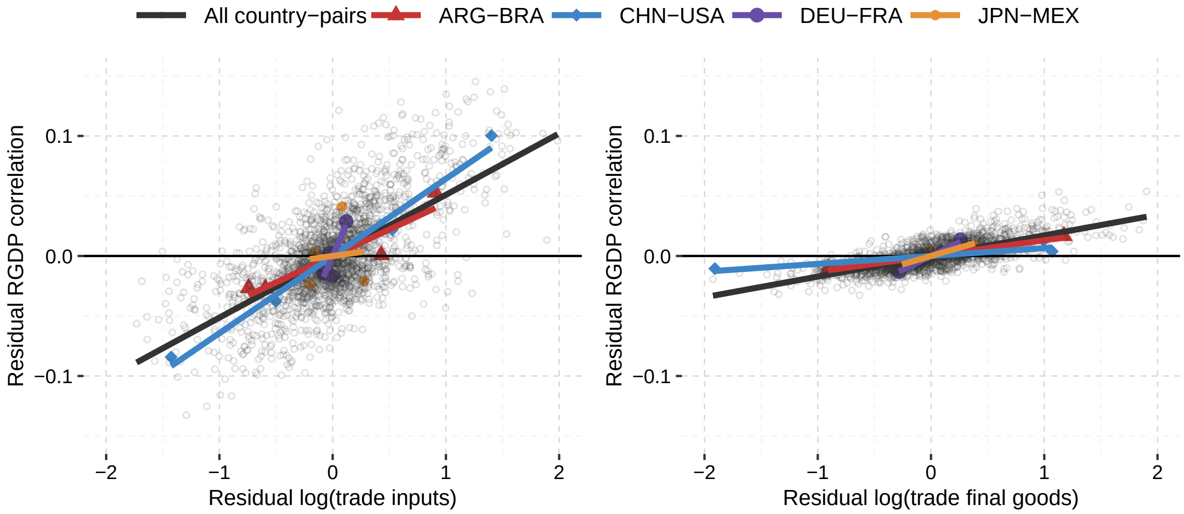
[26] A complementary approach to this common exposure term has recently been proposed in Avila-Montealegre and Mix (2020). Their analysis measures the exposure to correlated trade partners, which captures a possible high common exposure even for country-pairs that do not share the same partners.

[27] For each time-window experiment, we feed in the *exact same* TFP shocks over our 100 replications so that the only thing that changes accross time windows is the calibrated level of trade. In section 5.3, we also do a robustness test where the correlation of TFP shocks changes over time and is disciplined by TFP data in the PWT. In this case, the TC slope estimated with both country-pair and Time-Windows fixed effect is slightly below our baseline model's estimate.

the four time windows. For example, relative to the first time-window of our sample (1970-1979), the median bilateral trade intensity in intermediate goods and final goods increased by 100% and 185% in the last time-window (2000-2009). More details about our computation procedure can be found in Appendix A.

Consistent with the procedure used in our empirical exercise, we then estimate equation (30) and exploit within country-pair variation. We first focus on the relationship between measured real GDP correlations (corr $RGDP$) and bilateral trade intensities in intermediate inputs ($\text{Trade}_{ij}^{\text{input}}$) and final goods ($\text{Trade}_{ij}^{\text{final}}$). In Figure 1, we plot the residual relationship between bilateral trade in final and intermediate inputs and $RGDP$ synchronization, after controlling for country-pair fixed effects and change in the other good bilateral trade intensity (either final or intermediate goods). The model is found to capture well the high and significant correlation between real GDP comovement and bilateral trade in intermediate goods, while the effect of trade in final goods is found to be negligible.[28]

Figure 1. Model-based association between RGDP correlation and Trade intensities. Left chart shows Intermediate Inputs trade, right chart shows Final goods trade.



Note: residual relationship in the model after controlling for other covariates, i.e. country-pair fixed effects and the other trade intensities (either final or intermediate goods). The grey solid line reports the TC slope including all country-pairs. Selected country-pairs

The implied slope in our benchmark model is reported in Table 4 (1st row, columns (1)-(2)). Focusing on intermediate inputs, the model generates a significant and positive TC slope of 0.047, which represents 75% of the empirical estimates in Table 3.[29] To get a sense of this

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[28] In the robustness section 5.3, we show that a higher Armington elasticity for final goods aggregation can account for a negative slope between cross-country GDP correlations and trade in final goods.

[29] Using total bilateral trade intensity which captures the sum of input and final good trade, we find a slope of 0.051, comparable to the range of values $[4.8\% - 11\%]$ reported in the literature.

number, our point estimates suggest that a doubling of log input trade index generates an increase of pairwise real GDP correlation in our simulations of about 11%.

Table. 4. Quantitative assessment of the Trade Comovement Slope: Data versus Model.

Trade – comovement slope [a]	Based on $RGDP$			Based on PVA	
	Input (1)	*Final* (2)	slope input model/data	*Input* (3)	*Final* (4)
Data: CP & TW fixed effects	.063**	−.023			
1. Benchmark: IO link. + Markups + EM	.047***	.008***	.75	.009***	.006***
2. Model with IO link. + Markups	.021***	.001***	.33	.004***	.003***
3. Model with IO link.	.005***	.006***	.08	.005***	.006***

Notes: *p<0.1; **p<0.05; ***p<0.01. SE clustered on country-pairs. Results are robust to the inclusion of our $network_{prox}$ index. The trade indexes are calculated using $(T_{i\to j} + T_{j\to i})/(GDP_i + GDP_j)$.

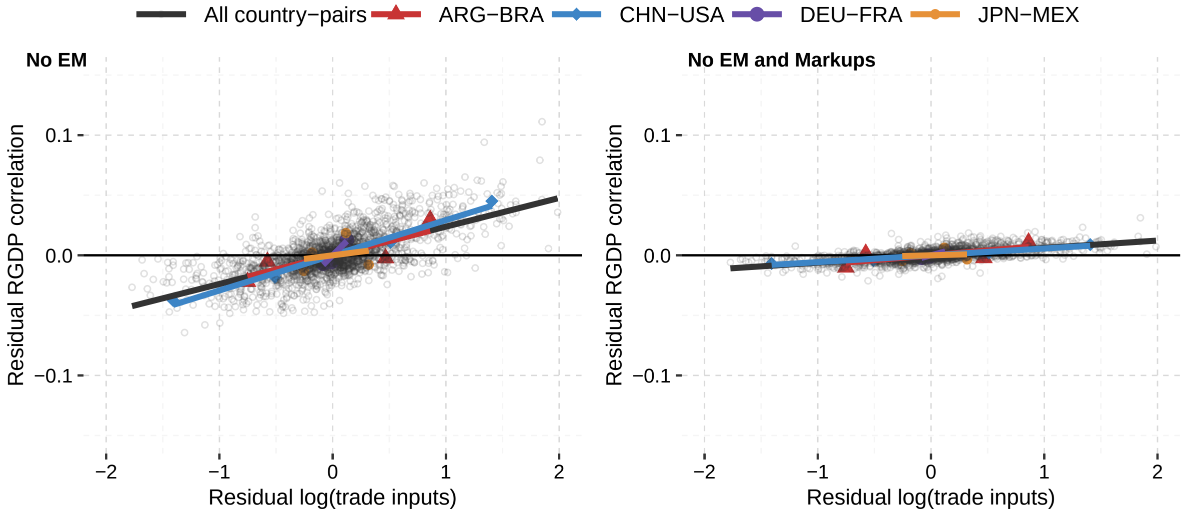
4.2.1 The Role of Extensive Margins and Markups

To understand the quantitative success of the model in replicating the large TC slope, we turn off one by one the key elements of the model: (i) movements along the extensive margin, and (ii) monopolistic competition. Figure 2 depicts the implied relationship between $RGDP$ and bilateral trade intensity in intermediate inputs under these two alternatives, with points estimates in Table 4. With markups but no extensive margin (EM) adjustments, the TC slope in intermediate inputs is 0.021 (Figure 2, left panel, and Table 4, 2^{nd} row). In a model with neither markups nor extensive margin adjustments (Figure 2, right panel, and Table 4, 3^{rd} row), the model delivers a virtually flat TC slope of 0.005. By comparing the implied slopes under those alternatives, a decomposition indicates that Input-Output links alone explain only 8% of the TC slope (=0.005/0.063) while the addition of markups alone and in combination with extensive margin channels increases the ratio to 33% and 75%, respectively.

4.2.2 The Importance of Measurement

As discussed in section 2, the introduction of price distortions and extensive margin adjustments generate a disconnect between measured real GDP ($RGDP$) and physical value added (PVA). In our simulations, the association between trade and the synchronization of physical value added is low, consistent with earlier findings in the literature. Columns (3)-(4) in Table 4 show that, for all model specifications, using the PVA measure consistently results in a negligible TC slope. As anticipated, without love of variety and markups (as indicated in the 3^{rd} row),

Figure 2. Decomposition of the Input Trade-comovement slope using alternative model specifications. Left chart shows a model without Extensive Margin adjustments, right chart shows a model with neither extensive margin nor markups.



Note: residual relationship in the model after controlling for other covariates, i.e. country-pair fixed effects and the other trade intensities (either final or intermediate goods).

there is no difference between the slopes obtained using PVA and $RGDP$. These results highlight the importance of defining real GDP in the model in a way that is consistent with data construction procedure: a researcher using PVA as a proxy for real GDP would mistakenly conclude that the model is not consistent with the data.

To sum up, our results show that adding intermediate inputs crossing multiple borders to an otherwise standard IRBC model is not sufficient to solve the TCP. In models with perfect competition, RGDP is equal to PVA, which reacts only modestly to foreign shocks. However, a model combining I/O linkages with markups and extensive adjustments in conjunction with statistically-consistent measured real GDP provides a quantitative solution to the Trade Comovement Puzzle.

4.3 Comparison to the Data and other Frameworks

We conclude the results section by discussing how our quantitative exercise compares to previous TCP studies. First, it is important to note that IRBC models in which the production function is expressed in value-added terms use PVA as a measure of real GDP. In the context of the TCP literature, papers such as Kose and Yi (2006) (equation 19, p.278) and Johnson (2014) (equation 10, p.47) evaluate the TC slope using PVA, while Burstein and Cravino (2015) (equation 6, p.188) use the double deflation method with base period prices. In such perfectly competitive model, however, our discussion in section 2 shows that $RGDP$ and PVA are equal. In what follows, we discuss in more details how our framework compares to two closely related investigations: Johnson (2014) and Liao and Santacreu (2015).[30]

The quantitative exercise in Johnson (2014) shows that a model with I/O linkages and real GDP measured as PVA cannot replicate the TC slope observed in the data. Our paper complements this finding by showing that adding markups and extensive margin adjustments, together with a measure of $RGDP$ using double deflation and base period prices, allows the model to generate a TC slope of an order of magnitude larger. There are several additional points worth mentioning here.

First, Johnson (2014) focuses on cross-sectional variations, and the TC slope obtained in simulations with correlated shocks reflects the fact that country-pairs with higher trade intensities also have more correlated TFP shocks. In our paper, we are interested in the endogenous relationship between trade and GDP comovement. We therefore control for bilateral correlation through country-pair fixed effects and focus on *within* country-pair change in GDP comovement following a change in trade intensities. Second, Johnson (2014) highlights that the TC slope is stronger when using gross output rather than GDP. As shown in Appendix A, our model also predicts this result, as gross output becomes more correlated due to the correlated use of intermediate inputs. Third, Johnson (2014)'s empirical results show that the cross-sectional TC slope is lower in the service sector relative to the good sector.[31] Although this is a feature we cannot directly compare in our model since we do not separate goods and services, we believe our framework can help understand his result. In section 2, we show that the share of intermediate input in production has an important role in generating co-movement. In the data, the share of input use is lower in the service sector than in the good sector. The median share of intermediate inputs in total cost for goods sectors is 65%, while the corresponding share for services is 46%. As such, we should expect a lower comovement in the service sector through the lens of our mechanism. Finally, Johnson (2014)'s analysis also gives rise to a puzzle our model cannot solve. When looking specifically at the service sector, his results show that $RGDP$ are highly correlated across countries, despite measured-TFP being uncorrelated. However, through the lens of our model, ceteris paribus, higher $RGDP$ comovement comes from higher comovement of productivity as measured by the Solow Residual. Hence, we believe that when analyzing the TC slope in the service sector, other factors should be considered.

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[30] We thank anonymous referees for suggesting a comprehensive comparison with these frameworks.

[31] Service sectors account for roughly 25% of total exports. As such, the aggregate TC slope is mostly driven by the goods sectors.

One candidate could be the introduction of dynamic trade elasticities (Drozd et al., 2021).

While most papers use *PVA* as a measure of real GDP, a notable exception is Liao and Santacreu (2015) who find that including an extensive margin component in a monopolistic competition model can account for part of the TC slope. In their simulations, real GDP is defined using a formula inspired by Burstein and Cravino (2015) which takes into account love of variety effects. This metric yields a TC slope of 18% of their empirical estimates (Table 8, p.276) and 22% of our estimates when using total trade intensities. As such, while they are able to explain a larger fraction of the TC slope relative to the previous literature, their simulated TC slope remains significantly below its empirical counterpart. The apparent discrepancy between Liao and Santacreu (2015) and our results highlight several elements worth to mention.

First, an important difference comes from the fact that real GDP in Liao and Santacreu (2015) does not use base period prices (see equation 47, p.276). Although gross output and intermediate input are deflated by their own price index, which results in "quantity-like" objects, these quantities are not valued using base period prices as in (29).[32] Second, imported input in their framework are used to produce a non-tradable final good. As a result, no imports are re-exported further, which limits the type of "network propagation" that are present in our model (see equations (24) and (27)) or in Johnson (2014). Overall, Liao and Santacreu (2015)'s framework incorporates the movements of the production frontier associated with extensive margin fluctuations but does not capture the overall measurement channel discussed here, which arises in presence of distorted base period prices in the measurement of real GDP.[33]

5 Further Investigations

5.1 Solow Residual, Profits and Trade

As shows in proposition 2, movements of the Solow Residual (which is often used as a proxy for productivity) reflect movements in the profits derived from imported inputs. It follows that an increase in intermediate input trade should be associated with an increase in the comovement of both Solow Residual and aggregate profits. We now test those predictions in the data and discuss their counterpart in the model.[34]

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[32] The importance of using base period prices in the definition of Real GDP has been highlighted prominently in Kehoe and Ruhl (2008), and is a key element of our proposition 2 in section 2.2.

[33] We thank particularly Ana Maria Santacreu for very insightful discussions regarding their results.

[34] Note that Kose and Yi (2006) and Liao and Santacreu (2015) study the relationship between measured TFP (measured as the Solow Residual) and bilateral trade in the cross-section. We add to their analysis by separating bilateral trade into intermediate inputs and final goods and use a panel estimation.

Data. We compute the Solow Residual (SR) based on a standard Cobb-Douglas production function. Profits are proxied by the Net Operating Surplus (NOS) measured by the OECD on a quarterly basis, which is adjusted for inflation using the consumer price index to eliminate any bias caused by inflation synchronization. Both measures are transformed using either HP-filter or log difference, before we compute pairwise correlations. We then study the relation between international trade (using $\text{Trade}_{ijb}^{\text{input}}$ and $\text{Trade}_{ijb}^{\text{final}}$) and those variables using:

$$\text{Corr V}_{ijb} = \beta_1 \ln(\text{Trade}_{ijb}^{\text{input}}) + \beta_2 \ln(\text{Trade}_{ijb}^{\text{final}}) + \mathbf{X}_{ijb} + \text{CP}_{ij} + \text{TW}_b + \epsilon_{ijb}^{V}, \tag{31}$$

where V refers to either SR or NOS. Results are gathered in Table 5. Consistent with our predictions, higher trade intensity in intermediate inputs tend to have a positive and statistically significant effect on the co-movement of measured productivity and profits.[35] Trade in final goods is found to have a negative and statistically insignificant effect. Moreover, our theoretical results in section 2 highlighted in propositions 1 and 2 suggest that controlling for SR correlation in specification (30) should capture both markup and love of variety effects. In online appendix OA3.2, we show that this control indeed results in a low and insignificant point estimate for trade in intermediate inputs.[36,37]

5.1.1 The Trade-Productivity Slope in the Model

We use our theoretical framework to investigate the above empirical finding. Again, productivity is measured using the Solow Residual: $SR_{it} = \log(RGDP_{it}) - \alpha \log(L_{it}) - (1-\alpha)\log(K_{it})$. Table 6 shows the resulting trade–productivity slope in the baseline model and versions of the model without extensive margins or markups.

The first row reveals that measured productivity correlation increases when country-pairs trade more intermediate inputs, and the magnitude is consistent with data. Without extensive margin adjustments or markups, productivity is only equal to technology shocks, which are identical in all simulations, thus the association between trade and productivity correla-

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[35] Note that the higher profit-comovement slope could be rationalized by the presence of pro-cyclical time-varying markups which further amplifies the link with trade-linkages. We thank an anonymous referee for this valid point.

[36] As discussed in Huo et al. (2020), unobserved factor utilization can create a measurement error for SR. In the online appendix OA3.2, we introduce an unobserved component in labor input leading to measurement errors in SR. Using model-based simulations, we show that such unobserved component leads to a downward attenuation bias in the slope between trade and $corr(SR)$. This finding implies that the positive and significant results obtained in the data using specification (31) may be a lower bound of the slope's true value.

[37] In the online appendix OA1.5, we also show that the comovement of labor input (as measured in the PWT) is not statistically associated with trade integration. While there might be issues with measuring labor input in the data, the absence of statistical association between trade links and measured labor input synchronization suggests that looking beyond factor supply comovement is important when looking at cross country RGDP correlation.

Table. 5. Trade, Measured Productivity (SR) and Net Operating Surplus (NOS)

	Corr SR$^{\text{HP}a}$		Corr ΔSR^{a}		Corr NOS$^{\text{HP}b}$		Corr ΔNOS^{b}	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
ln(Trade$^{\text{input}}$)	0.060**	0.069***	0.049*	0.052**	.371***	.175*	.288***	.250***
	(0.028)	(0.027)	(0.025)	(0.025)	(.104)	(.095)	(.083)	(.086)
ln(Trade$^{\text{final}}$)	-0.003	-0.029	-0.005	-0.020	$-.085$	$-.105$	$-.100$	$-.157^{**}$
	(0.025)	(0.026)	(0.023)	(0.023)	(.079)	(.073)	(.062)	(.079)
Country-pair fixed effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Time window fixed effects	No	Yes	No	Yes	No	Yes	No	Yes
N	2,224	2,224	2,224	2,224	364	364	364	364
Within R^2	0.111	0.218	0.110	0.185	.104	.263	.121	.273

Notes: *p<0.1; **p<0.05; ***p<0.01. In parenthesis: std. deviation. SE clustered on country-pairs. Controls include third indexes and trade unions. Results are robust without third indexes.

[a] Data for SR are constructed using the PWT 9.1 using total employment and measured capital. We use 2/3 for the labor share. Due to missing data, we lose some country-pairs relative to our benchmark sample.

[b] Data for Corr NOS are measured at quarterly frequency, for 16 consecutive quarters in time-windows of 4 years. Due to data limitation, our sample covers 1999Q1-2014Q4. Trade flows are taken from OEC data. Result are robust without the Great Recession (2007-2010 time-window) and the inclusion of additional controls: dummies for trade unions and similarity indexes.

tion disappears. Additionally, when productivity is measured as the Solow Residual of PVA (defined as: $SR^{PVA-based}it = \log(PVA_{it}) - \alpha \log(L_{it}) - (1-\alpha)\log(K_{it})$), there is no disconnect between technology and measured productivity.

This discussion underscores the importance of measuring real GDP in a way that is consistent with actual data construction procedures. Our simulations show that the median correlation of measured productivity (SR) is about 0.29, while the median correlation of technology shocks is 0.09, illustrating that using measured productivity as a target for technology would overestimate actual shock correlation between countries.

Table. 6. Association between trade and SR comovement: Data versus Model.

Trade – Productivity comovement slope:^{a}	SR based on *RGDP*			SR based on *PVA*	
	Input (1)	*Final* (2)	slope input data/model	*Input* (3)	*Final* (4)
Data: CP & TW fixed effects	.069***	$-.029$			
1. Benchmark: IO link. + Markups + EM	.043***	.002***	.62	$-.000^{***}$	.000***
2. Model with IO link. + Markups	.015***	$-.002^{***}$	.22	.000*	.000***
3. Model with IO link.	.000	$-.000$	.00	.000	.000***

Notes: $*$p<0.1; **p<0.05; ***p<0.01. SE clustered on country-pairs. Results are robust to the inclusion of our $network_{prox}$ index. The trade indexes are calculated using $(T_{i\to j} + T_{j\to i})/(GDP_i + GDP_j)$. Productivity is measured as the Solow Residual using either $RGDP$ or PVA as a measure for real value added.

5.1.2 A Focus on the Role of Markups and Profits in the Model

We now examine the significant role of markups in the cross-country correlation of profits and productivity as seen in Table 5.[38] Our baseline calibration uses a lower-tier elasticity of substitution of $\sigma = 4.0$. Table 7 presents our simulation-based TC slope using alternative values for the markups. We first test the implication of higher ($\sigma = 3.5$) and lower ($\sigma = 5.0$) markups in the 2nd and 3rd row, respectively. As expected, increasing markups from 25% to 33% significantly amplifies the association between input trade and real GDP synchronization, while the response using PVA (in the last two columns of Table 7) remains insignificant.

We then introduce heterogenous market power across countries. Specifically, we simulate the model with heterogenous σ_i using direct markup estimates from De Loecker and Eeckhout (2018) (DLE) (4th row).[39] The implied elasticity σ_i ranges between 3.3 and 11.1. Interestingly, adding heterogeneous markups does not substantially affect the TC slope, which suggests that accounting for cross-country heterogenous markups does not change the aggregate strength of international propagation in our model.

Table. 7. The role of price distorsions and heterogenous markups. [a]

Trade – GDP comovement slope:		based on $RGDP$			based on PVA	
	Elasticity σ	*Input*	*Final*	slope input data/model	*Input*	*Final*
Data: CP & TW fixed effects		.063**	−.023			
1. Baseline	4.0	.047***	.008***	.75	.009***	.006***
2. Low markups	5.0	.039***	.011***	.62	.012***	.008***
3. High markups	3.5	.060***	.008***	.95	.009***	.007***
4. Heterogenous markups	$[3.3, 11.1]$	.042***	.007***	.67	.009***	.006***

Notes: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$. Results are robust to the inclusion of our $network_{prox}$ index. The simulations are based on the exact same sequence of shocks and variations of trade indexes used in the benchmark.

Finally, our findings offer a possible explanation for the wide range of TC slopes seen in the literature, which can range from 4.8% to 11% depending on the country and time frame. The model can account for this heterogeneity by considering variations in market power across countries and over time.

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[38] In section OA1.8 of the online appendix, we further validate the role of markups in generating a link between terms of trade and real GDP fluctuations.

[39] The sample that we use from their estimates includes 29 countries from 1980 to 2016. The values are reported in the online Appendix OA3.1. We assign the mean value when data are not available. In an earlier version, we also used OECD STAN's database and construct the Price Cost Margin (PCM) as an estimate of markups within each industry, which measures the difference between revenue and variable cost. Results were similar to those reported using DLE data.

5.2 Extensive Margin fluctuations and Real GDP Comovement in the Data

We finally investigate the role of extensive margin adjustments in generating the observed association between trade and real GDP comovement. We use the Hummels and Klenow (2005) (HK) decomposition and investigate the relation between the average and the volatility *within* each time window of the Extensive Margin (EM) and Intensive Margin (IM) of bilateral input trade intensities and real GDP comovement. We use a panel of the same 35 countries and analyze trade data classified as intermediate inputs under the SITC (rev. 2, 4-digits), obtained from the NBER United Nations Trade Data (1971-2000) and UN COMTRADE data (2001-2010).

Using the HK decomposition, we construct the EM and IM of trade in intermediate inputs for each directed pair of country $(i \to j)$ in a given year t. The *Rest-of-the-World*, indexed k, is taken as a reference country. The EM is defined as a weighted count of varieties, indexed s, of intermediate inputs exported from country j to country i relative to those exported from country k to country i, i.e. $\text{EM}_{ijt} = \sum_{s \in \Omega^I_{ijt}} T^{\text{input}}_{k \to i,t}(s) / \sum_{s \in \overline{\Omega^I}_t} T^{\text{input}}_{k \to i,t}(s)$, where Ω^I_{ijt} is the set of observable categories in which j has a positive shipment to i and $\overline{\Omega^I}_t$ is the set of all categories of intermediate inputs exported by the reference country. If all categories are of equal importance and the reference country k exports all categories to i, then the extensive margin is simply the fraction of categories in which j exports to i. The corresponding IM is the ratio of nominal shipments from j to i and from k to i in a common set of intermediate goods Ω^I_{ijt}, i.e. $\text{IM}_{ijt} = \sum_{s \in \Omega^I_{ijt}} T^{\text{input}}_{j \to i,t}(s) / \sum_{s \in \Omega^I_{ijt}} T^{\text{input}}_{k \to i,t}(s)$. Note that the product of the two measures provide a measure of the overall trade from j to i relative to the overall trade from k to i. Finally, since those measures are not symmetric within a country-pair we sum, for each country pair (i, j), the IM_{ijt} and EM_{ijt} from i to j and from j to i, and normalize this by the sum of GDP. We denote the corresponding variables $\widehat{\text{EM}}_{ijt}$ and $\widehat{\text{IM}}_{ijt}$.

We compute the *within* time-window (indexed b) average and standard deviation (std) of EM and IM and test:

$$\text{Corr GDP}_{ijb} = \beta_1 \ln(\widehat{\text{EM}}_{ijb}) + \beta_2 \ln(\widehat{\text{IM}}_{ijb}) + \text{CP}_{ij} + \text{TW}_b + \epsilon^m_{ijb} \tag{32}$$

$$\text{Corr GDP}_{ijb} = \beta_1 \ln(std(\widehat{\text{EM}})_{ijb}) + \beta_2 \ln(std(\widehat{\text{IM}})_{ijb}) + \text{CP}_{ij} + \text{TW}_b + \epsilon^s_{ijb} \tag{33}$$

Results are reported in Table 8. Using specification (32) in columns (1) and (3), we recover a result in line with Liao and Santacreu (2015) who use an IV estimator instead of a panel estimation: the correlation between the level of the extensive margin of trade in intermediate inputs and real GDP comovement is positive and significant. In contrast, the intensive margin of trade is found not significantly related with GDP comovement. However, it is worth noting that our theory suggests that what matters is the *variation*s in the number of traded input varieties, rather than the level of the extensive margin, as it is *fluctuations* of the extensive margin that impact GDP comovement. Using specification (33), we test this by relating GDP comovement to the standard deviation of extensive and intensive margin movements. Results in columns (2) and (4) show a positive and significant correlation between larger fluctuations along the extensive margin and higher GDP comovement, while the intensive margin shows no significant relationship.[40]

Table. 8. Real GDP correlations and the margins (EM and IM) of intermediate inputs trade.

	Corr $\text{GDP}_{ijt}^{\text{HP filter}}$		Corr ΔGDP_{ijt}	
	(avg measure) (1)	(std measure) (2)	(avg measure) (3)	(std measure) (4)
EM	0.061^{*}	0.062^{***}	0.053^{*}	0.075^{***}
	(0.031)	(0.021)	(0.031)	(0.022)
IM	-0.008	-0.026^{**}	0.004	-0.017
	(0.021)	(0.011)	(0.020)	(0.010)
CP + TW fixed effects	Yes	Yes	Yes	Yes
N	2,220	2,220	2,220	2,220
Within R^2	0.094	0.099	0.108	0.115

Notes: $^{*}p<0.1$; $^{**}p<0.05$; $^{***}p<0.01$. In parenthesis: std. dev. SE clustered on country-pairs. The number of observation drops relative to Table 3 due to some country-pairs with not enough intermediate inputs linkages.

5.3 Robustness and Alternative Specifications.

As our result is quantitative in nature, it is important to check the validity of our results under alternative model specifications and calibrations. In Table 9 of Appendix A, we examine the model's ability to produce the TC slope with different parameter values related to the elasticity of labor supply, international trade costs, capital adjustment costs, and the Armington elasticity in the CES good aggregation. In those experiments, our results are virtually unchanged. Our results also hold under complete financial markets (which constitutes the other extreme relative to financial autarky), Cobb-Douglas utility, and different variations of trade linkages. We investigate alternative correlations of technology shocks (Z), and find that our results are little changed when using uncorrelated shocks, negatively correlated shocks, and time-window varying TFP correlations. Finally, we also check the sensitivity of the results under alternative

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[40] This result is particularly striking given that most of the variations in trade at business cycle frequency is explained by variations along the intensive margin. In the online appendix OA1.7, we further investigate the role of the extensive and intensive margins of trade using an alternative dataset using a direct measure of the number of firms and find that the extensive margin of trade is positive and significant.

RGDP measurement: the ideal Fisher index to construct base period prices, gross output, and no correction for variety.

6 Conclusion

This paper analyzes the relationship between international trade and business cycle synchronization across countries, with a focus on improving the mapping between real GDP in the data and its counterpart in standard macroeconomic models. We show that real GDP, as constructed by statistical agencies, is not equal to the theory-consistent "physical value added". This disconnect appears when the base period price used to value imported input does not reflect their marginal product, for example in presence of markup and love of variety. With those ingredients, real GDP fluctuations are not only tied to movements of technology and factor supply, but can also fluctuate as a result of changes in imported input usage.

Quantitatively, the presence of markups and love of variety delivers a strong link between input trade and business cycle comovement, with a magnitude in line with empirical estimates, offering a solution for the *Trade Comovement Puzzle*. Conversely, model-based simulations show that trade is much less associated with the synchronization of physical value added. We finally confirm the predictions based on the data. First, higher trade in intermediate inputs is associated with an increase in the bilateral correlation of both real GDP and productivity. Second, higher trade in intermediate input is also associated with synchronized profits. Third, real GDP is sensitive to fluctuations in the number of varieties imported, implying that the extensive margin of trade plays an important role in business cycle synchronization.

To conclude, this paper seeks to draw attention to real GDP measurement in macroeconomic models and how it should be compared with its data counterpart. In the context of the *Trade-Comovement Puzzle*, IRBC models that generate weak cross-country propagation properties in terms of physical value-added can actually feature strong propagation in terms of real GDP. More generally, recognizing that real GDP fluctuations are not only tied to physical value added movements could be a promising research avenue for business cycle analysis.

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A Robustness and Alternative Specification

Our results are robust to a number of alternative specifications, as presented in table 9.

Table. 9. Model-based simulations: sensitivity analysis. [a]

Trade – GDP comovement slope	Change	based on *RGDP*	
		Input	*Final*
1. Baseline	-	.047***	.008***
A. Model parameter values and specification			
2. High Frisch elasticity	$\nu = .25$	.060***	.015***
3. Low Frisch elasticity	$\nu = .75$	.040***	.004***
4. Iceberg costs	+10%	.047***	.008***
5. No trade imbalance	$\mathcal{T}_i = 0,\ \forall i$	.046***	.005***
6. Lower adjustment cost	$\psi = 4.00$	.055***	.012***
7. Alternative CES elasticity	$\rho^F = 1.50$	.045***	$-.008$***
8. Cobb Douglas utility	*see text*	.043***	.005***
9. Complete markets	*see text*	.042***	.016***
10. Uniform increase in trade linkages	*see text*	.044***	.008***
B. Productivity process $cov(Z_i, Z_j)$			
11. Uncorrelated Z	$= 0, \forall i \neq j$	.046***	.008***
12. Negatively correlated Z	$= -.05, \forall i \neq j$	.043***	.007***
13. Time-window varying correlated Z	$= \widehat{cov}(SR_{it}, SR_{jt})$	.040***	.000
C. Measurement			
14. Fisher index	*see text*	.048***	.008***
15. "Welfare-based" measure	*see text*	.067***	.034***
16. No base period price in numerator	*see text*	.016***	$-.014$*
17. No correction for variety in price indices	*see text*	.050***	.015***

[a] The simulations are based on the exact same sequence of shocks used in the baseline model.

A.1 Computation procedure

Solving the model with 35 countries implies more than 10000 equations. To reduce computation time, we partition our sample of 35 countries into five groups of seven countries, and simulate our model by combining two of those groups plus a composite of the rest-of-the-world – which means we do simulations with a total of 14 countries + Rest of the World. We repeat this exercise for all possible pairs of groups and thereby recover all the country-pairs of our sample of 35 countries. The results are similar if we use other partitions (for example five groups of seven countries, or 3 groups of 9 countries + a group of 8, etc...), confirming that such simplification offers a good approximation of the model including all the country-pairs at the same time.

A.2 Parameter Values and Model Specification

Parameters. The Frisch elasticity has a significant impact on the magnitude of the TC slope. In row 2, we use a Frish of 4 ($\nu = 0.25$), which is the value in Johnson (2014), and obtain a significantly larger TC slope, while a lower Frish of 1.5 ($\nu = 0.75$) yields a smaller TC-slope in row 3. Regardless of the initial level of trade frictions and trade imbalances, $\{\tau_{ij}, \mathcal{T}_{i,t}\}$, increasing trade proximity is associated with the same reaction for real GDP comovement, as can be seen in rows 4 and 5. In rows 6, we vary ψ and find that lower adjustment costs results in more volatile real value added and investments, which magnifies the TC slope. A higher Armington elasticity of $\rho_F = 1.25$ in the CES final good aggregation (row 7) can rationalize the negative and insignificant slope for trade in final goods. The use of a Cobb-Douglas utility specification (row 8), such that $U(C_{i,t}, L_{i,t}) = (C_{i,t}^{\eta}(1 - L_{i,t})^{1-\eta})^{1-\sigma}/(1-\sigma)$ with η calibrated to generate an aggregate hours worked of $1/3$ and $\sigma = 2$, does not change the main message.

Alternative Financial Markets. Our benchmark specification assume financial autarky. We verify if the results of our quantitative model hold under complete financial markets, which can be thought as the other extreme modelling assumption. We assume that there are complete contingent claims dominated in units of one of the countries' tradable final good. Let s_t denote the state of an economy in period t, with transition probability density $f(s_{t+1}, s_t)$. We denote $B_i(s_{t+1})$ denote the country i's holdings of a one-period state-contingent bonds, paying off one unit of the numeraire good in state s_{t+1}, and let $b(s_{t+1}, s_t)$ be the price of that security in state s_t at date t. Furthermore, these state-contingent bonds are in zero net supply in all states: $\sum_i B_i(s_{t+1}) = 0$. In this case, the consumer budget constraint is given by:

$$P_{i,t}^{F}(C_{i,t} + \mathcal{I}_{i,t}) + \int b(s_{t+1}, s_t) B_i(s_{t+1}) ds_{t+1} = r_{i,t} K_{i,t} + w_{i,t} L_{i,t} + B_i(s_t) \tag{34}$$

Consumers choose $\{C_{i,t}, L_{i,t}, K_{i,t+1}\}$ and asset holdings $\{B_i(s_{t+1})\}$ given prices and initial asset endowments $\{B_i(s_0)\}$ to maximize equation (12). Results are provided in Table 9 (row 9). We find that the TC slope is just a touch smaller under complete markets and the main message of the paper remains unchanged.

Alternative trade variations. In their seminal paper, Kose and Yi (2006) examine the impact of uniform shifts in trade linkages across country-pairs. In row 10, the study examines the impact of uniformly increasing all bilateral trade linkages by the median increase from 1970 to 2000, and finds that there is no significant change compared to the baseline model which utilized actual variations.

A.3 Productivity process

As argued above, the Solow Residual (SR) is a bias estimate for technology shocks Z in the model. To investigate how the calibration of the technology shocks Z impacts our results, we perform several robustness tests.

We first simulate the model under the counterfactual assumption that technology shocks are uncorrelated across countries and set the off-diagonal elements of the covariance-variance matrix to zero (i.e. $cov(Z_{i,t}, Z_{j,t}) = 0, \forall i \neq j$), in row 11 of Table 9. In this case, the TC slope is almost unchanged. Interestingly, even with uncorrelated Z shocks, the average correlation of Solow Residual SR is 0.15, which highlights important international propagation in the model.

Second, we simulate a version of our model with identical and negatively correlated Z shocks in row 12. The TC-slope within country pairs remains high. In row 13, we then use the observed evolution of the correlation of TFP shocks in the PWT (denoted $\widehat{cov}(SR_{i,t}, SR_{j,t})$) to calibrate the hypothetical evolution of Z shocks. In this version with time-varying correlation of shocks, the estimation of the TC slope requires the inclusion of Time Windows fixed effects, and the results do not alter our main conclusion.

A.4 Alternative GDP Measurement

Base period pricing. The baseline uses steady-state prices as base period prices. However, it's important to note that in practice, base period prices are typically constructed using the Fisher index. Intuitively, the Fisher index is a geometric average between two base period pricing methods, alternatively using $t-1$ (known as the Laspeyres Formula) and t prices (known as the Paasche Formula). The Laspeyres Formula uses period $t-1$ prices to compute real GDP growth such as:

$$RGDP_{i,t}^{Laspeyres} = RGDP_{i,t-1}^{Laspeyres} \cdot \frac{\widehat{\mathcal{P}_{i,t-1}^{F}} \, \frac{Y_{i,t}}{\widehat{\mathcal{P}_{i,t}^{F}}} + \sum_j \widehat{\mathcal{P}_{i,j,t-1}} \, \frac{T_{i\to j,t}}{\widehat{\mathcal{P}_{i,j,t}}} - \sum_j \widehat{\mathcal{P}_{j,i,t-1}} \, \frac{T_{j\to i,t}}{\widehat{\mathcal{P}_{j,i,t}}}}{Y_{i,t-1} + \sum_j T_{i\to j,t-1} - \sum_j T_{j\to i,t-1}} \tag{35}$$

The Paasche Formula uses period-t prices to compute real GDP growth such that:

$$RGDP^{Paasche}_{i,t} = RGDP^{Paasche}_{i,t-1} \cdot \frac{Y_{i,t} + \sum_j T_{i\to j,t} - \sum_j T_{j\to i,t}}{\widehat{\mathcal{P}^F_{i,t}} \frac{Y_{i,t-1}}{\widehat{\mathcal{P}^F_{i,t-1}}} + \sum_j \widehat{\mathcal{P}_{i,j,t}} \frac{T_{i\to j,t-1}}{\widehat{\mathcal{P}_{i,j,t-1}}} - \sum_j \widehat{\mathcal{P}_{j,i,t}} \frac{T_{j\to i,t-1}}{\widehat{\mathcal{P}_{j,i,t-1}}}} \tag{36}$$

Finally, the Fisher Formula is a geometric average of the Laspeyres and Paasche formula:

$$RGDP^{Fisher}_{i,t} = \left(RGDP^{Laspeyres}_{i,t}\right)^{0.5} \left(RGDP^{Paasche}_{i,t}\right)^{0.5} \tag{37}$$

As expected, our results hold using different measures of base period prices, as shown in row 14. Both our baseline metric (using steady state prices for base period prices) and the Fisher Index reveal that markups and love of variety significantly contribute to the strong TC slope. The fact that results are quantitatively similar when using the Fisher index vs. steady-state base period prices is intuitive: in our model, fluctuations happen around the steady state, which means that realized prices are never "*very far*" from steady state prices. The Fisher index is a geometric average of two metrics that use period-t and period-$t-1$ prices as base period (Laspeyres and Paasche formula), and each of these two metrics is not far from the simplified construction that simply uses steady state prices. This observation highlight that our argument relies quantitatively on the presence of base period prices in the construction of real RGDP, but the exact decision of *which base period price to use* (steady state prices, or period-t prices, or period $t-1$ prices) is quantitatively not very important. To reinforce this point, row 16 presents the results obtained using a metric that does not use base period prices in the numerator. In such a case, the association between comovement and trade is significantly smaller.

Welfare based measure and Gross Output. Recently, Burstein and Cravino (2015) proposed an alternative welfare-based measure for valuing the gains from trade. Although this measure is not directly linked to observed RGDP, Liao and Santacreu (2015) showed that it is more closely related to trade intensities when an extensive margin channel is present. To verify that this is indeed the case in our simulations, we construct a "Welfare-Based" index defined as: $WB_{i,t} = Y_{i,t}/P^F_{i,t}$. The results in row 15 confirm that the association between the comovement of this measure and trade links is stronger than for RGDP. Note that this measure is akin to Gross Output and our result is therefore in line with Johnson (2014), who observed that Gross Output tends to be more correlated with import movements.

Correcting for variety effects. Finally, we emphasize the significance of accurately defining prices with regards to variety effects – a point which is not new, and has been discussed precisely in the literature. Citing Ghironi and Melitz (2005) (Section IV, page 880): "*Under CES product differentiation, it is well-known that price indices can be decomposed into components reflecting average price and product variety. [...] The average price correspond much more closely to empirical measures such as the CPI.*" What is the consequence of not taking into account such correction? As a last experiment on measurement, we show in row 17 that using ideal price indices in the definition of RGDP (instead of correcting price indices for the variety effect) yields a TC slope which is higher than our benchmark. This observation highlights that it is important to correct for varieties to not over-estimate the performance of the model.